

# Physics of Virus Self-Assembly and Virus-like Biological Nanostructures

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UNIVERSITÉ  
DE MONTPELLIER



Established in 1289  
(Medical School since 1150)



**In the frame of the Program  
“IPOLS - International Physics  
of Living Systems”**



**- NSF (USA) + CNRS & INSERM (France)**

*Laboratory of Excellence “NUMEV”:  
Modeling for Life Sciences*



**3 health & biotechnology programs**

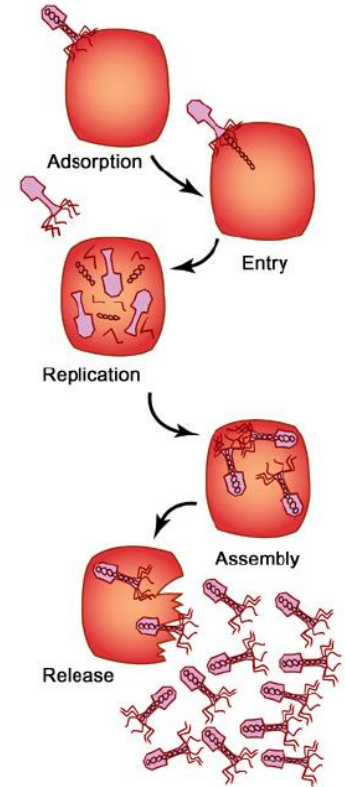
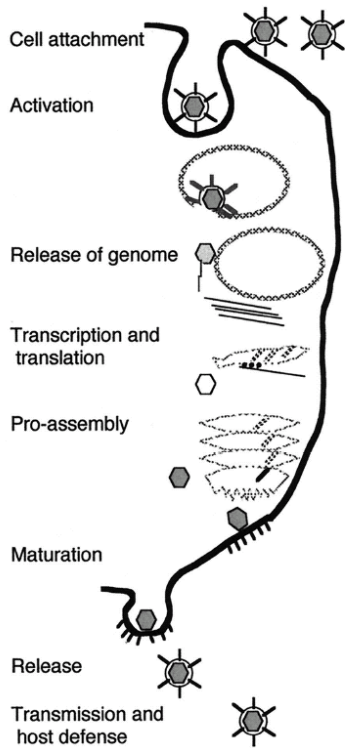
- Struggle against emergent infectious diseases
  - Controlled drug delivery
  - Bio-inspired nanotechnology

# Viruses, alive or not ?

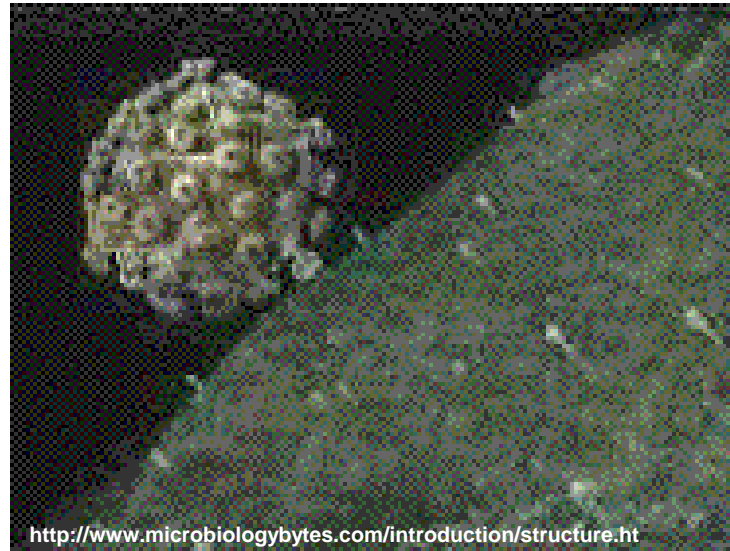
**All viruses require a host cell to replicate**

**Follow some basic pattern**

- Deliver viral genomic material into host cell
- Subvert cell's biosynthetic machinery into producing new viral particles
- New virus particles self-assemble in the infected cell
- New virus particles leave infected cell to infect others

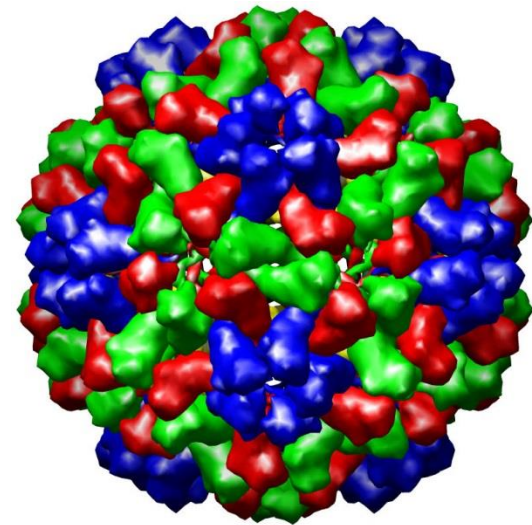
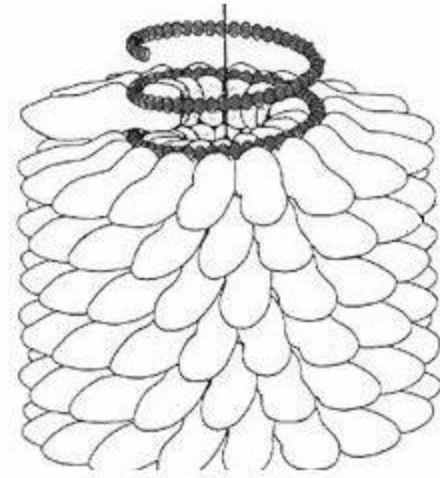


Lytic cycle



# Virus structure

- Genomic material
  - DNA or RNA (both “+” and “-”)
  - single- or double-stranded
  - linear or circular
  - one or several copies
- Capsid – Protective protein shell
  - regular organization
  - high symmetry
  - made of many identical subunits
  - enveloped with lipid membrane or not
- Basic shape
  - helical rod-like
  - spherical
- Typical size
  - 30 ÷ 50 nm
  - nearly 1000 nm for some big ones





# Exceptional shape viruses

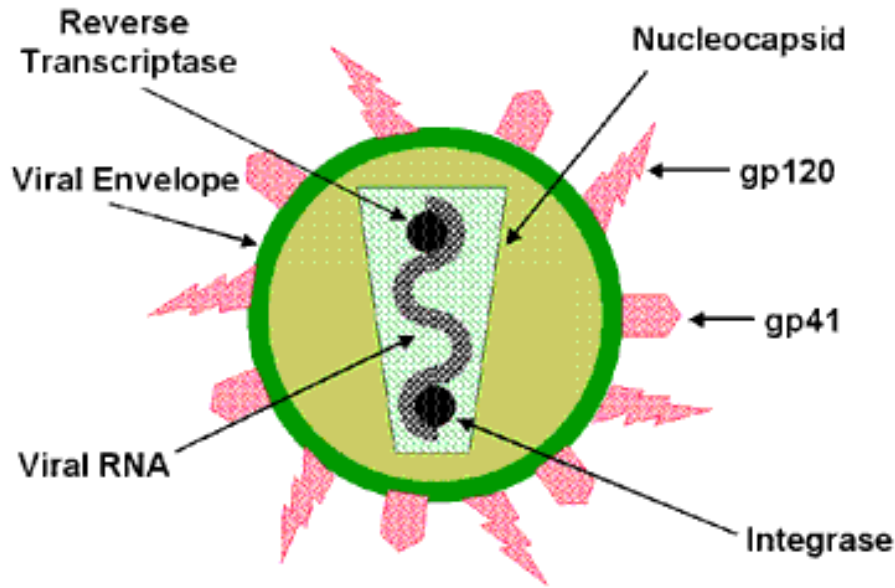
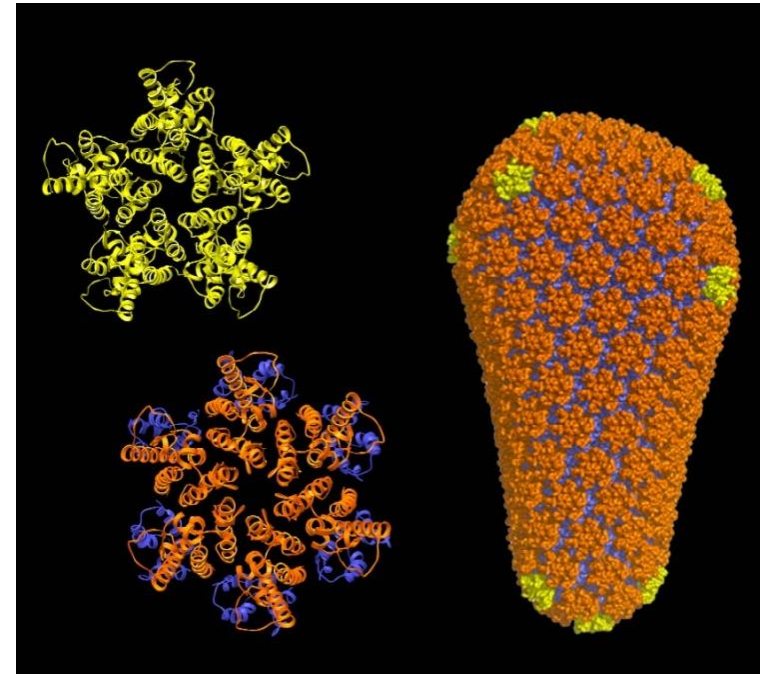


Figure 1. **Human Immunodeficiency Virus**

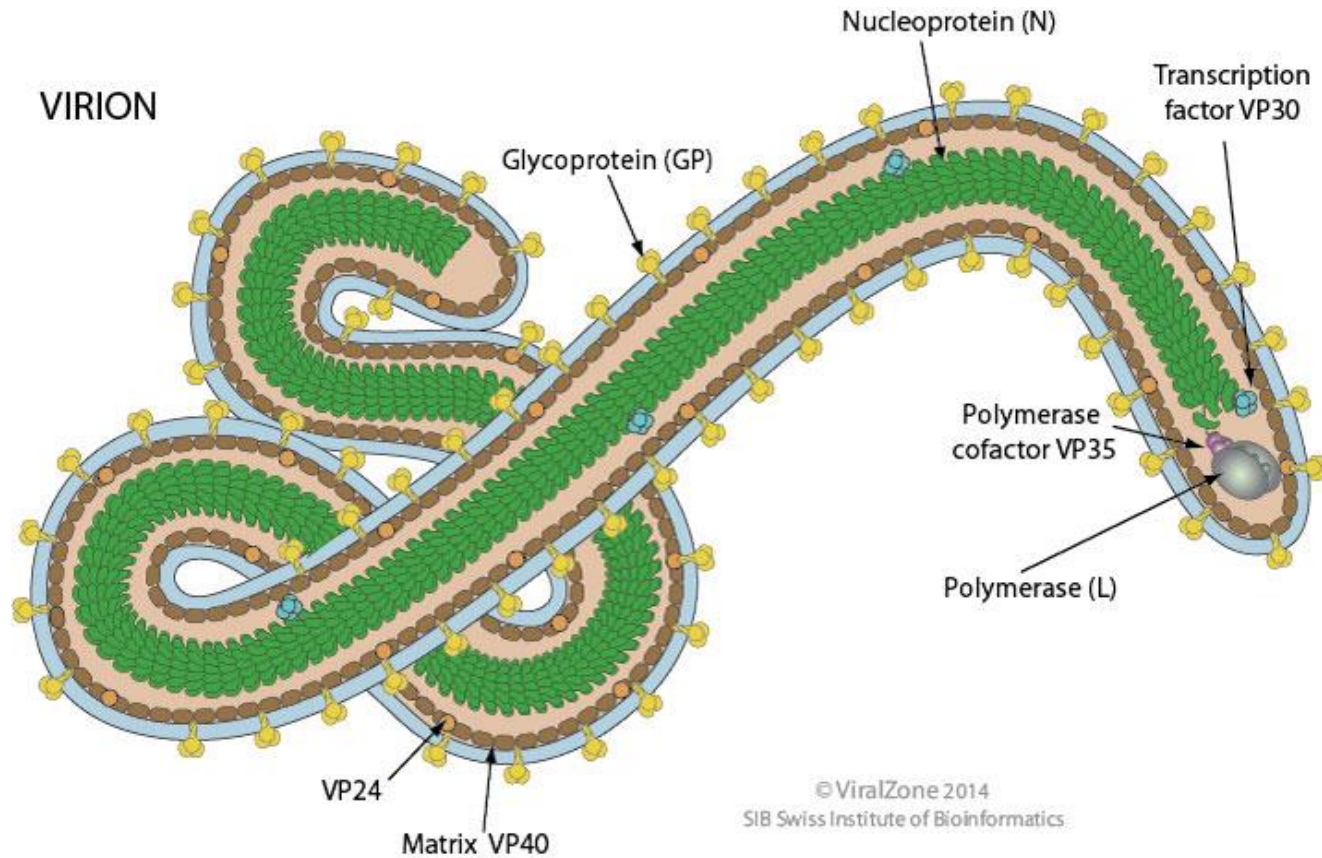


O. Pornillos *et al.*, *Nature*, **469**, 424 (2010)

The capsid of HIV is a *conical* shell, with *continuously varying* “lattice” curvature,

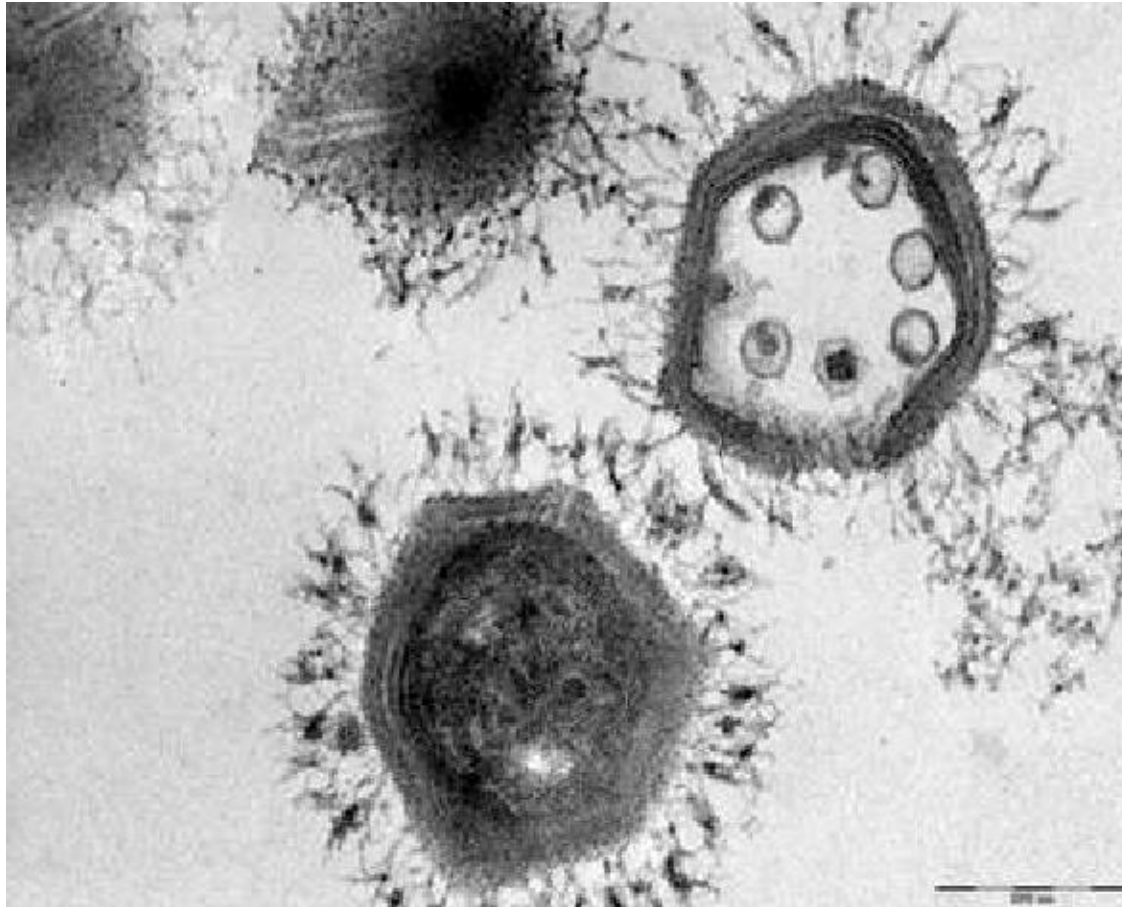
# Exceptional shape viruses

## Filoviridae ( Ebola virus)



Filamentous: compact and flexible . Mean diameter is about 80nm.

# Exceptional size viruses



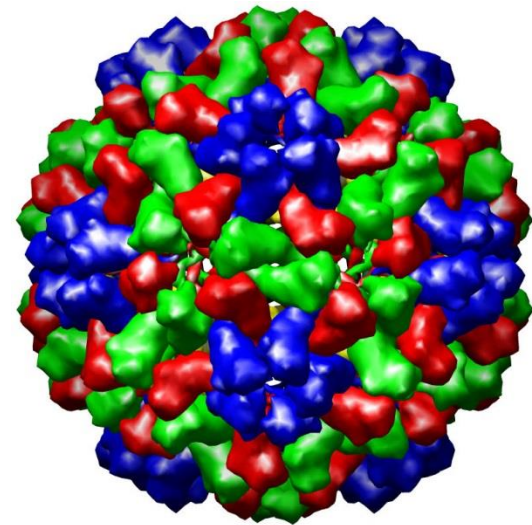
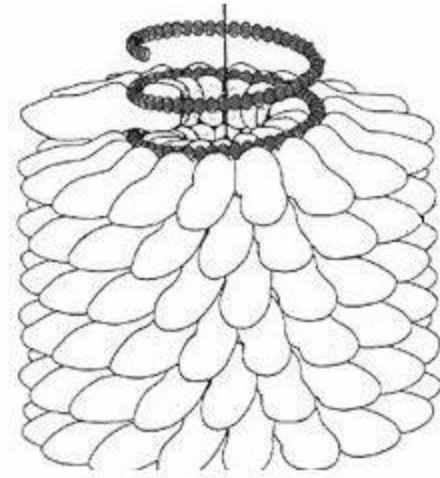
© CNRS Photothèque/URMITE/IRD 198 / RAOULT Didier,  
ESPINOSA Leon, CAMPANA Bernard

Giant Mamavirus infecting amoeba  
Mamavirus itself is infected by small Sputnik viruses

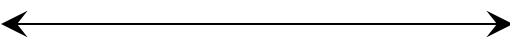
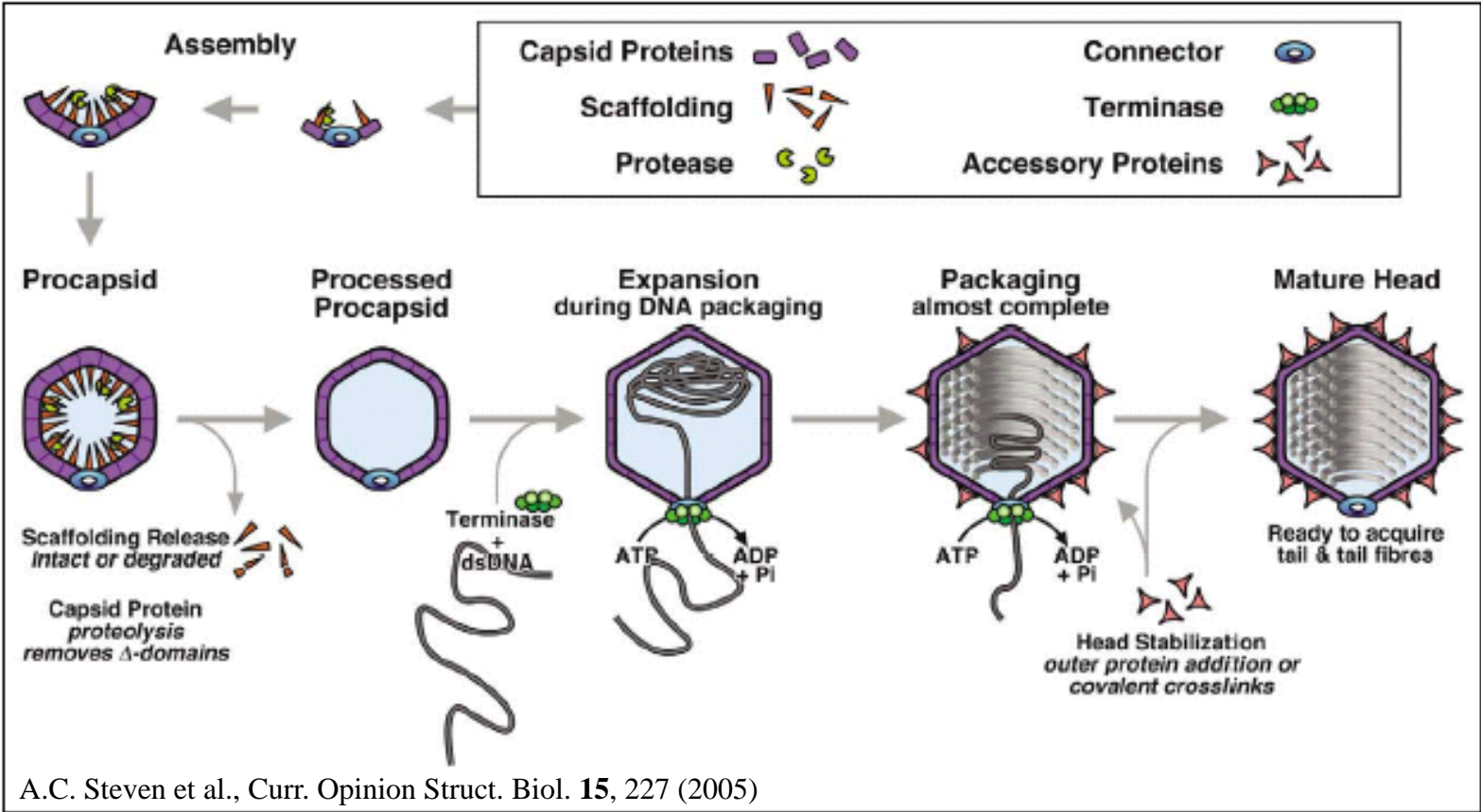


# Virus structure

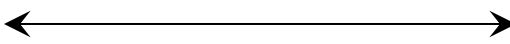
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  - regular organization
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  - made of many identical subunits
  - enveloped with lipid membrane or not
- **Basic shape**
  - helical rod-like
  - **spherical**
- **Typical size**
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# Virus Self-Assembly Process: Physicist's View



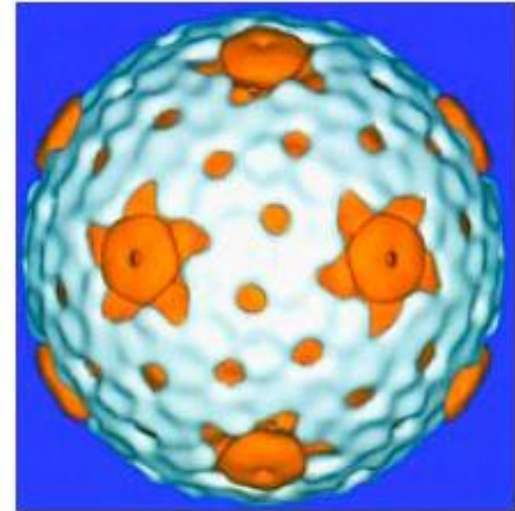
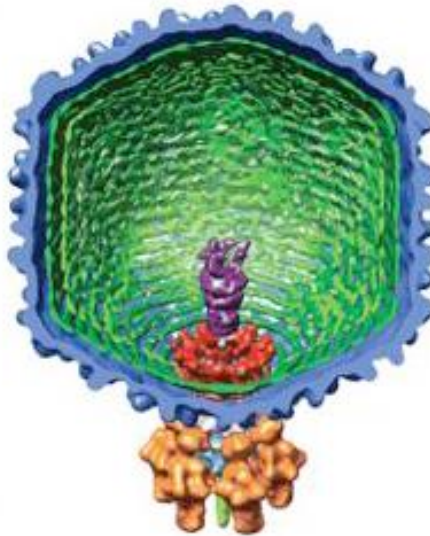
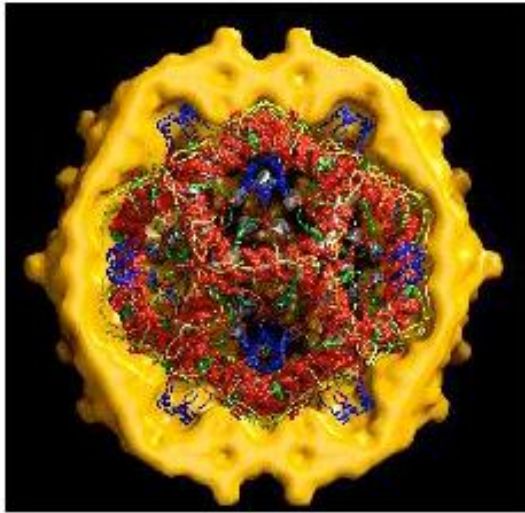
Procapsid Assembly  
from the Isotropic Solution



Procapsid-to-Capsid  
Transformation during Maturation

Main control parameters: protein concentration, ionic force, pH-level

# Genome packaging mechanisms



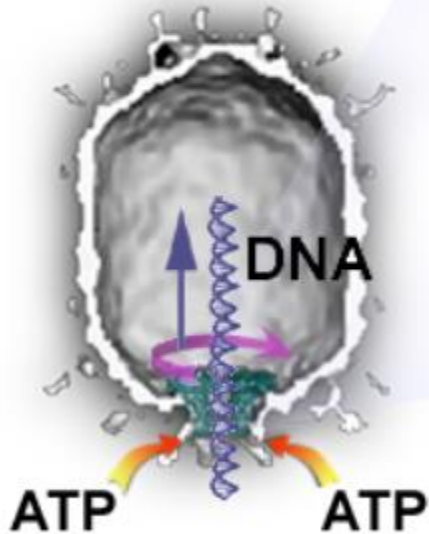
RNA Virus capsids assemble around their RNA genome.

Flock House Virus with its RNA structured as a dodecahedral cage.

dsDNA bacteriophage, packages its genome into a pre-assembled capsid through a portal.

# DNA packing in bacteriophages

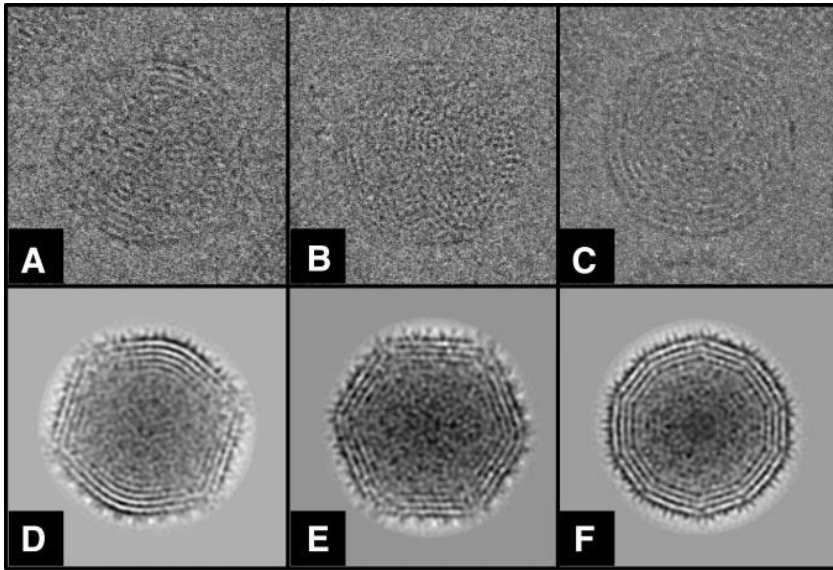
Empty Virus Shell



DNA Packaging Motor

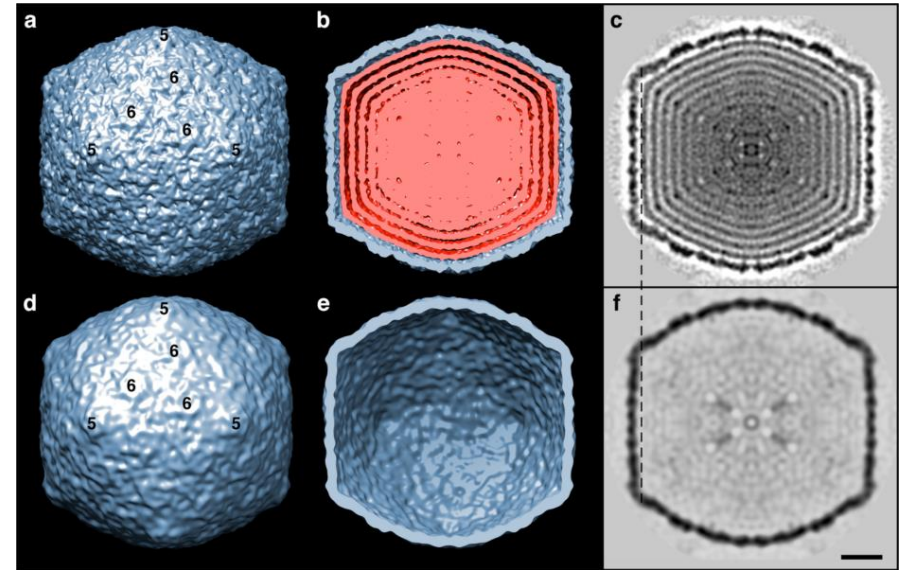


# DNA packing in bacteriophages



P22 phage

Z. Zhang *et al.*, *J. Mol. Biol.* **297**, 615 (2000)



HK97 phage

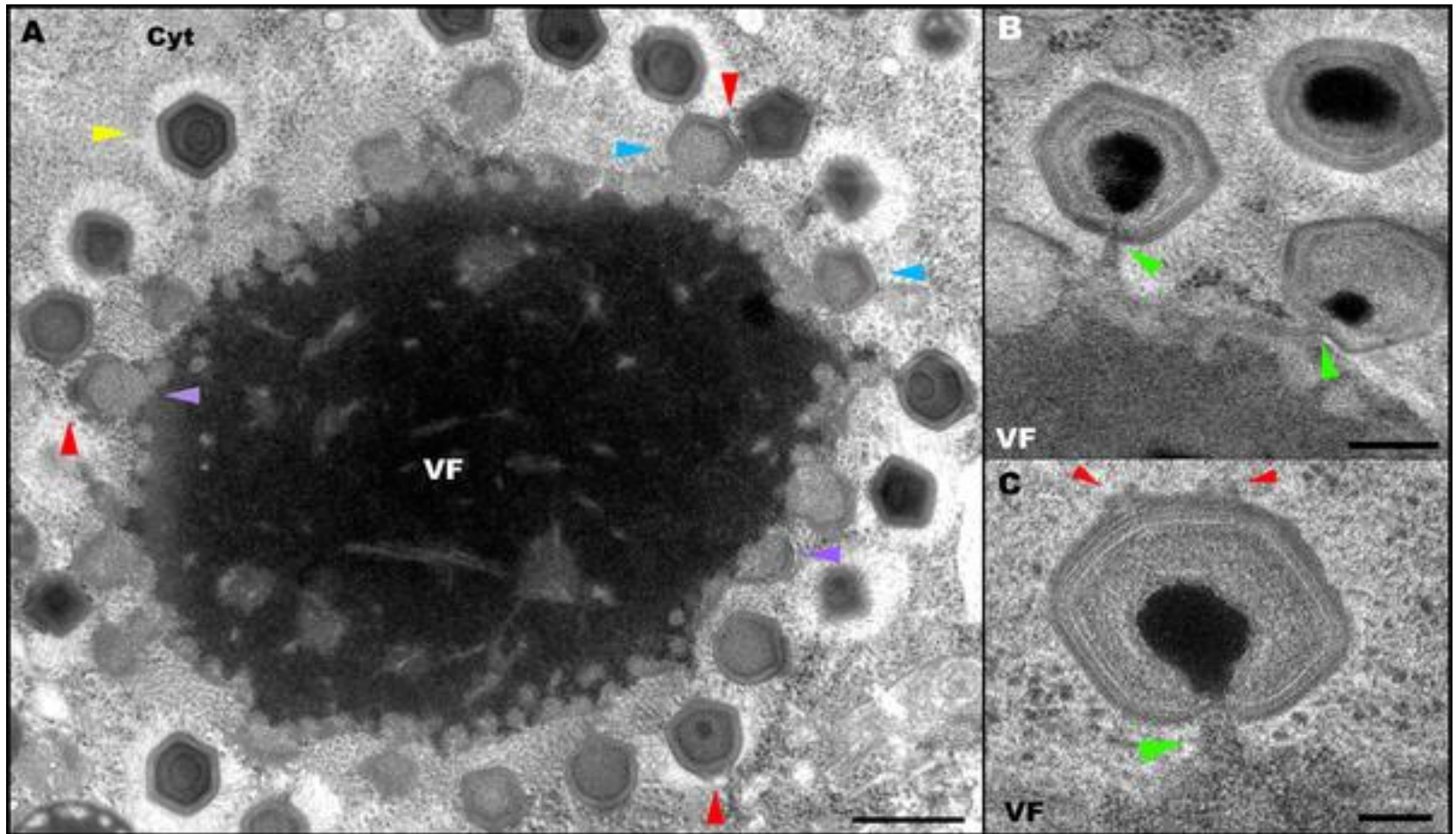
R.L. Duda *et al.*, *J Mol Biol.* **391**, 471 (2009)

Nested shells of packaged DNA or compact structures akin to hexagonal lattice

Outwards pressure exerted by the DNA  $\Delta P \approx$  up to 50 atm; enormous stock of elastic energy used then for genome injection.

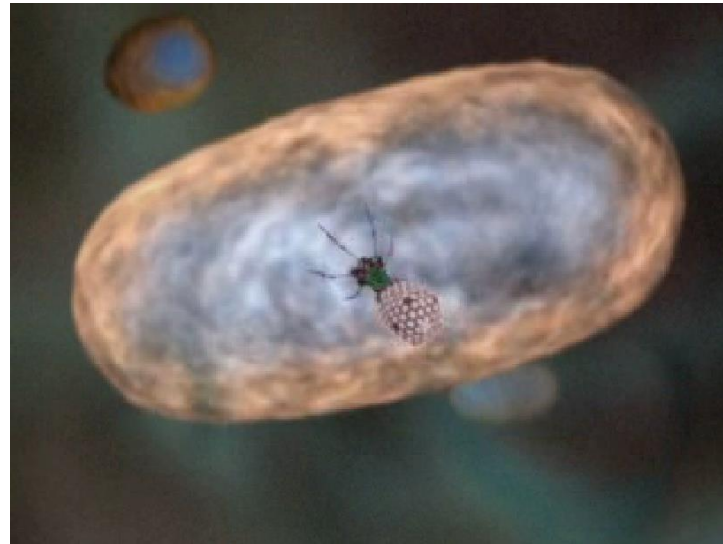
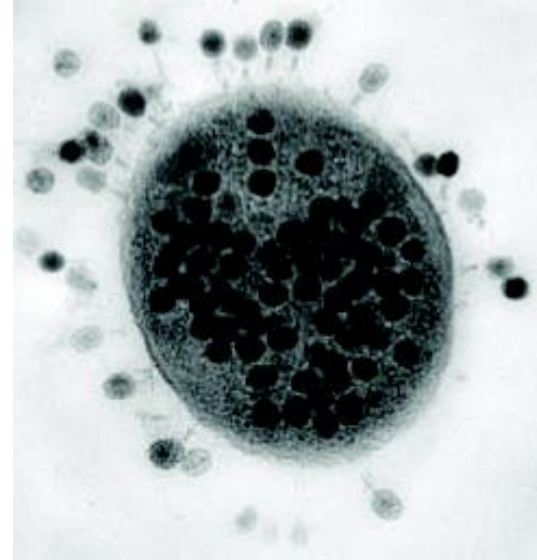
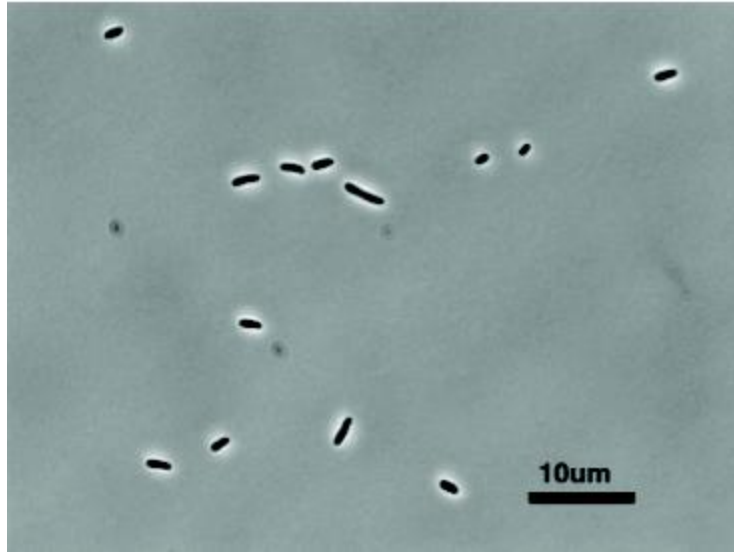


# Intracellular Viral Factories and DNA Packaging in Mimivirus



Zauberman N, Mutsafi Y, Halevy DB, Shimoni E, et al. (2008), PLoS Biol 6(5): e114

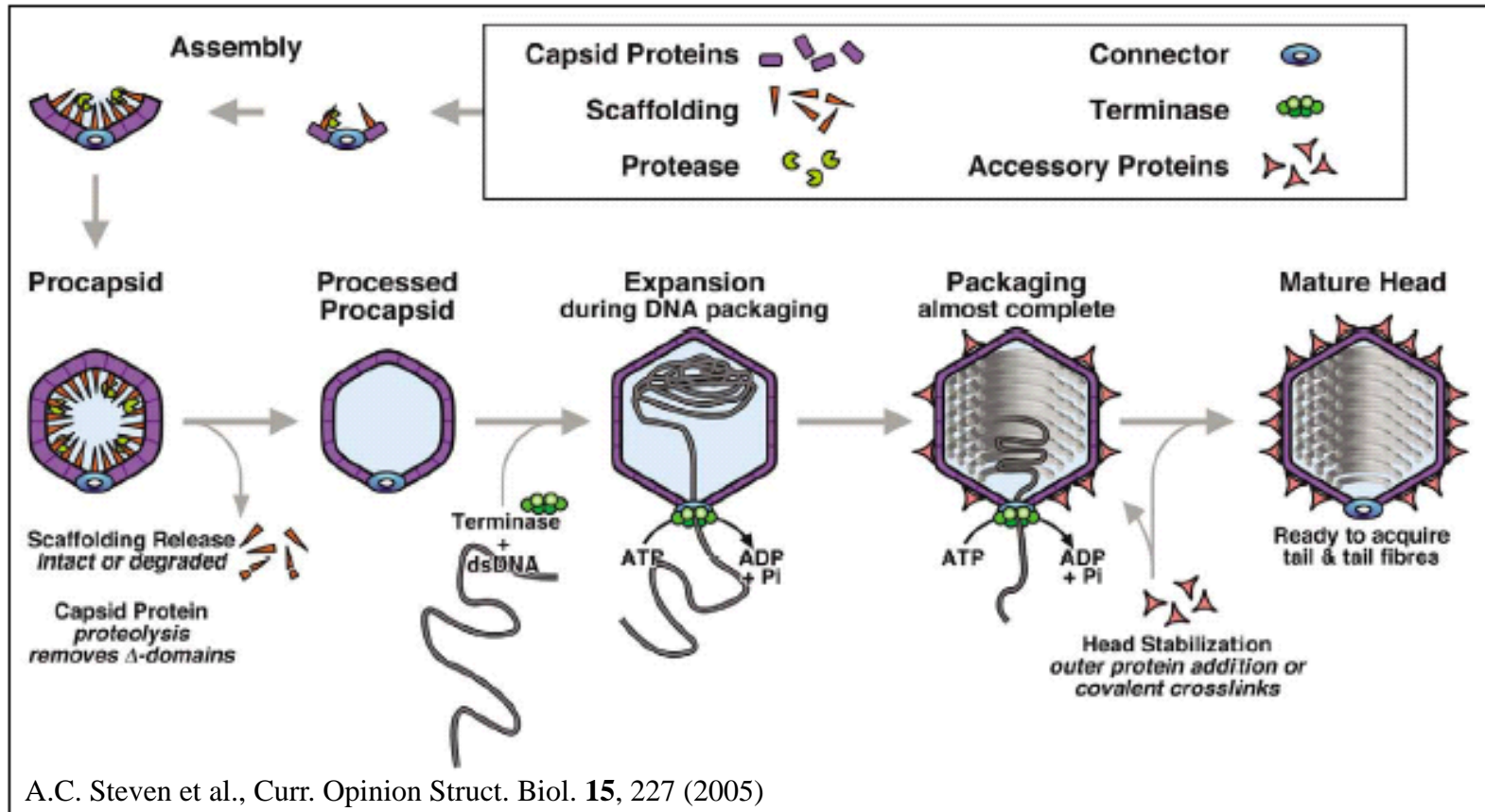
# DNA ejection into infected bacteria



# Drilling machine using commensurate shape transition



# Symmetry and Topology of Viral Capsids and their relation to the Self-Assembly



Procapsid Assembly  
from the Isotropic Solution

Procapsid-to-Capsid  
Transformation = Maturation

Main control parameters: protein concentration, ionic force, pH-level



# Top-down approach in Condensed Matter Physics

Systems characterized by :

- regular organization
- high symmetry
- made of many identical subunits

Basic principles :

- relation between phenomenological thermodynamics and symmetry of the system
- order parameter notion, representations of symmetry groups
- **reliable structural data**

Simple systems :

- crystallization process (including molecular crystals)
- solid-solid phase transitions

More complex systems :

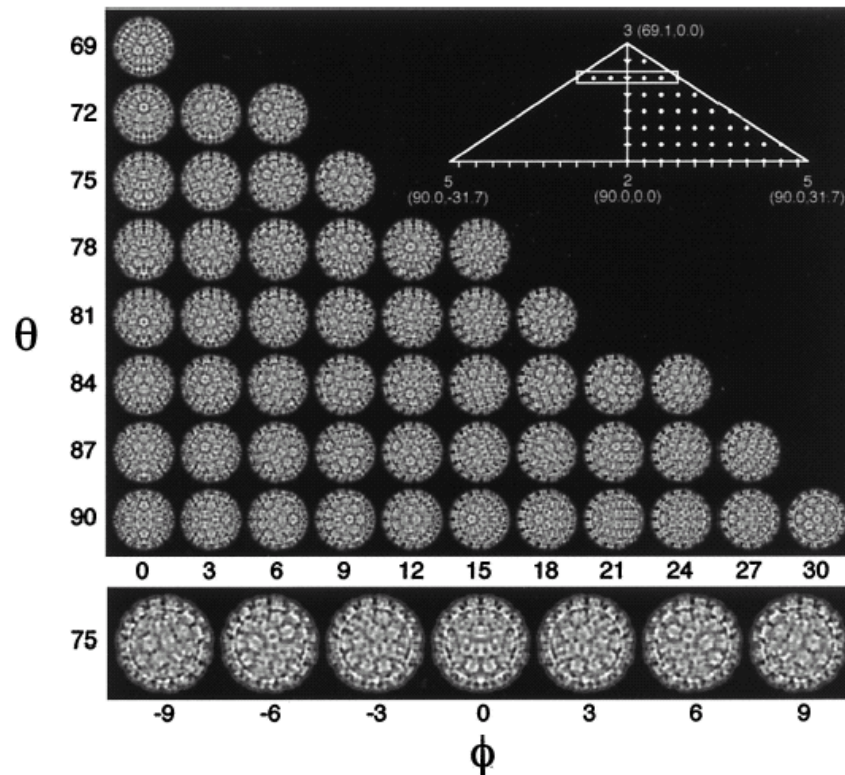
- ferroelectrics
- ferromagnetics
- superconductors
- liquid crystals



# Structural Data

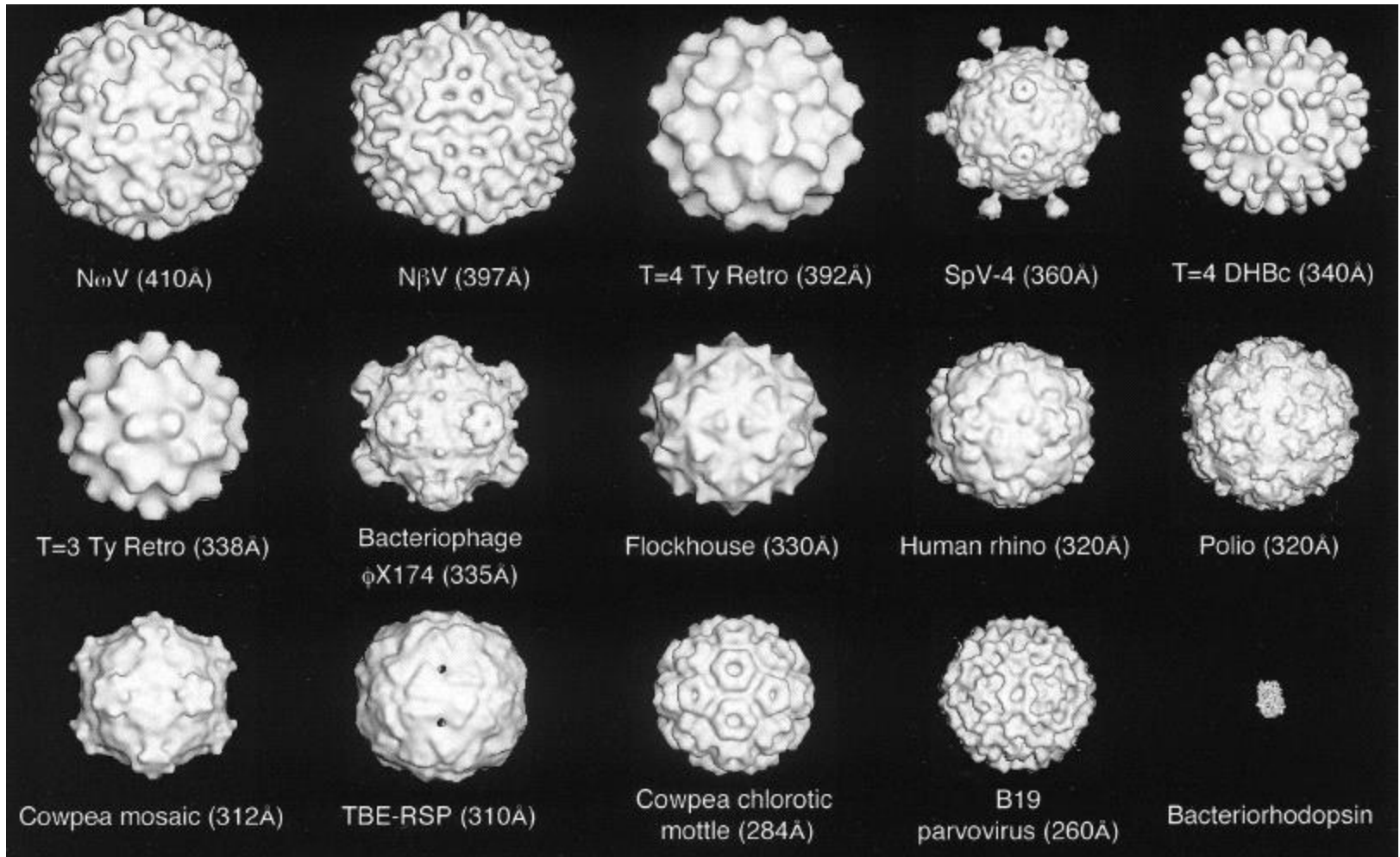
Symmetry of viral capsids make them uniquely well-suited to structural methods: X-Ray & CryoEM

⇒ Viruses are the largest aggregates of biological macromolecules whose structures have been determined at high resolution



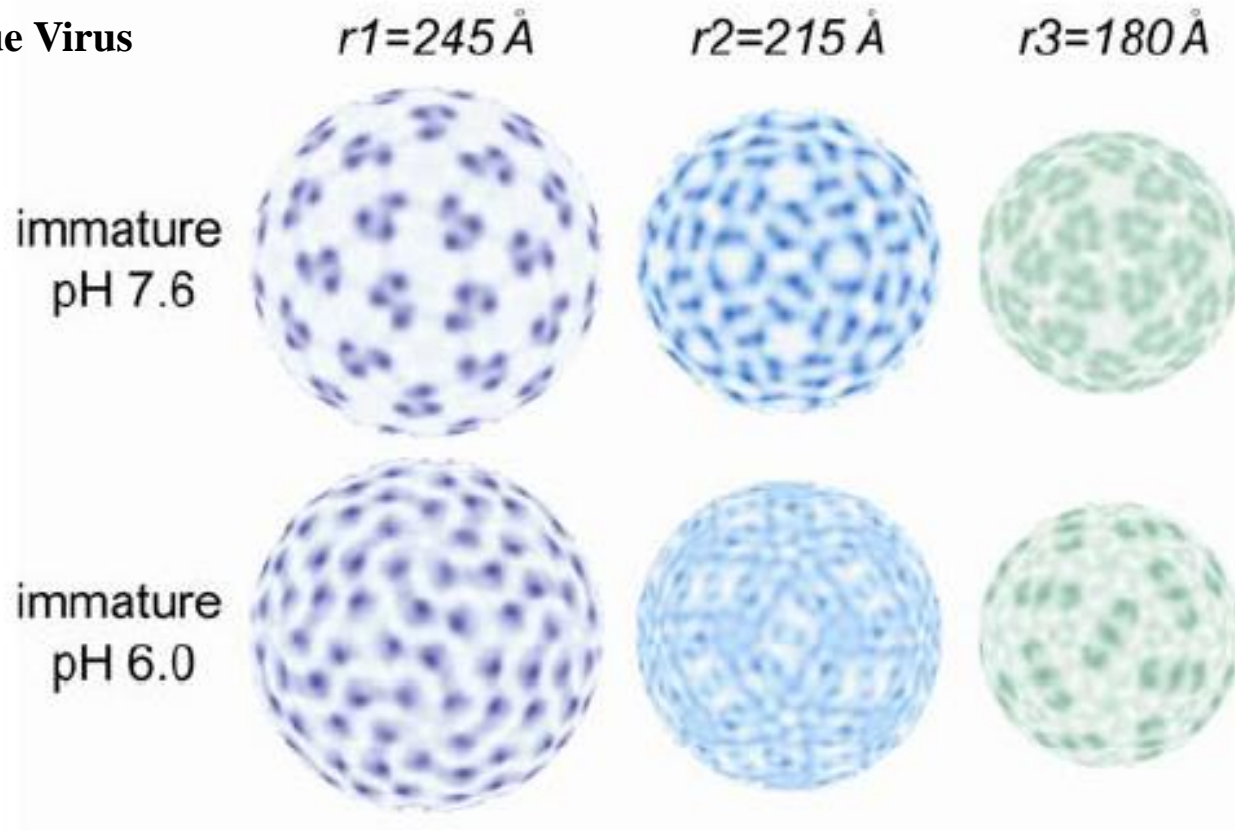
T.S. Baker et al.  
*Microbiol. Mol. Biol. Reviews* **63**, 862 (1999)

# Structural Data



# Detailed Structural Data

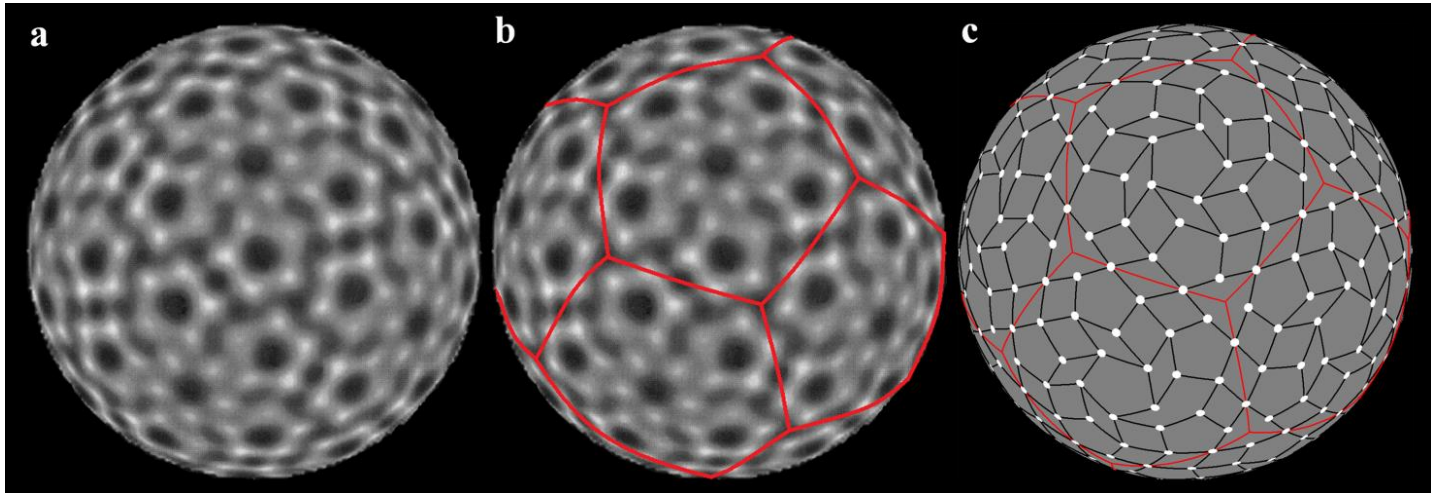
Ex.: Dengue Virus



Radial sections of virion density cryo-reconstructions and projection of the density on the sphere of maximal density

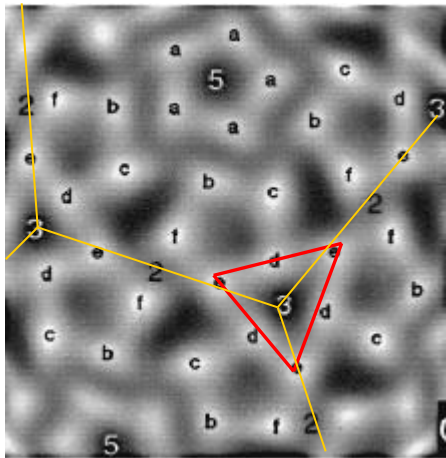
Available from cryoEM community

# Detailed Structural Data



Ex.: **Bovine Papilloma Virus**

O.V. Konevtsova et al.  
Phys. Rev. Lett. 2012



- ***Quantitative*** structure analysis
- ***Physical models***
- Direct ***comparison*** with the data and predictions

# Basics : How it started

**F.H.C. Crick & J.D. Watson (1956), D.L.D. Caspar & A. Klug (1962) :**

## ***Basic principles of virus structure***

Key insight : Limited volume => Limited genome size

=> only few sorts of proteins of limited size

***typically one protein for capsid formation***

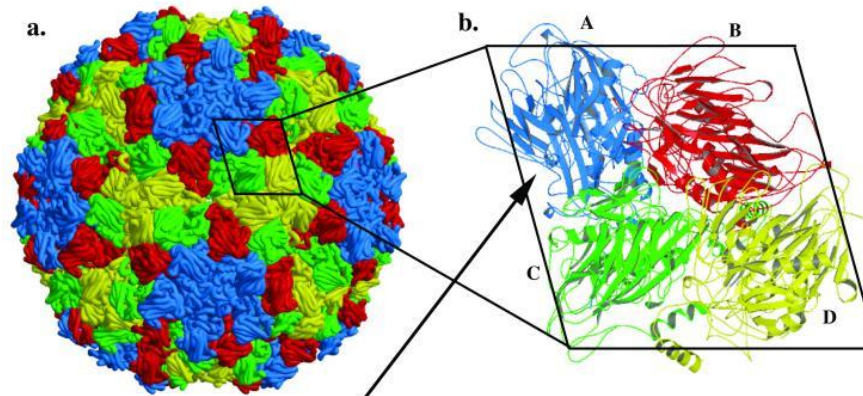
Proteins are ***“identical”*** subunits in ***“identical”*** environments

=> First: irreducible 3D symmetry, then: ***icosahedral*** symmetry

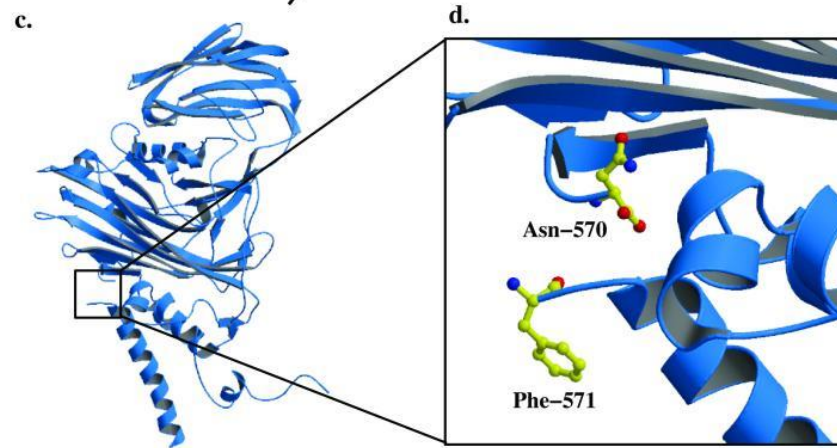
Thermodynamics :

Self-Assembly is a ***process akin to crystallization***





Energy of a Phase Transition



Energy of a Protein Conformation

Energy of a Covalent Bond

Different levels of Matter Organization and Energy Scale Separation Principle

Transition : isotropic solution of individual proteins  $\rightarrow$  unconventional 2D "crystal" with non-trivial spherical topology

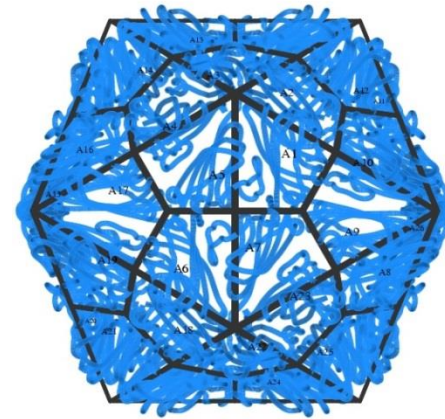
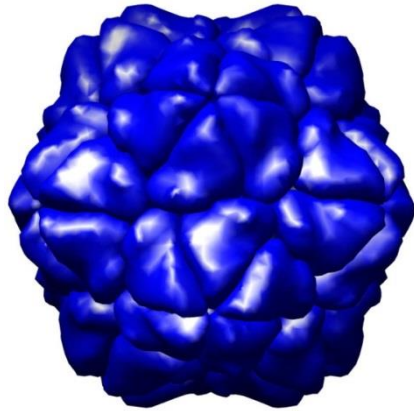
## I. Selection Rules for Positions of Protein Centers of Mass based on :

- Spherical Topology
- Asymmetry of individual proteins
- Global icosahedral symmetry of the Assembly

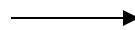
## II. Relation Structure – Assembly Thermodynamics

# Role of protein asymmetry

Moments of mass distribution. Simplest asymmetry property - chirality



Asymmetric Protein



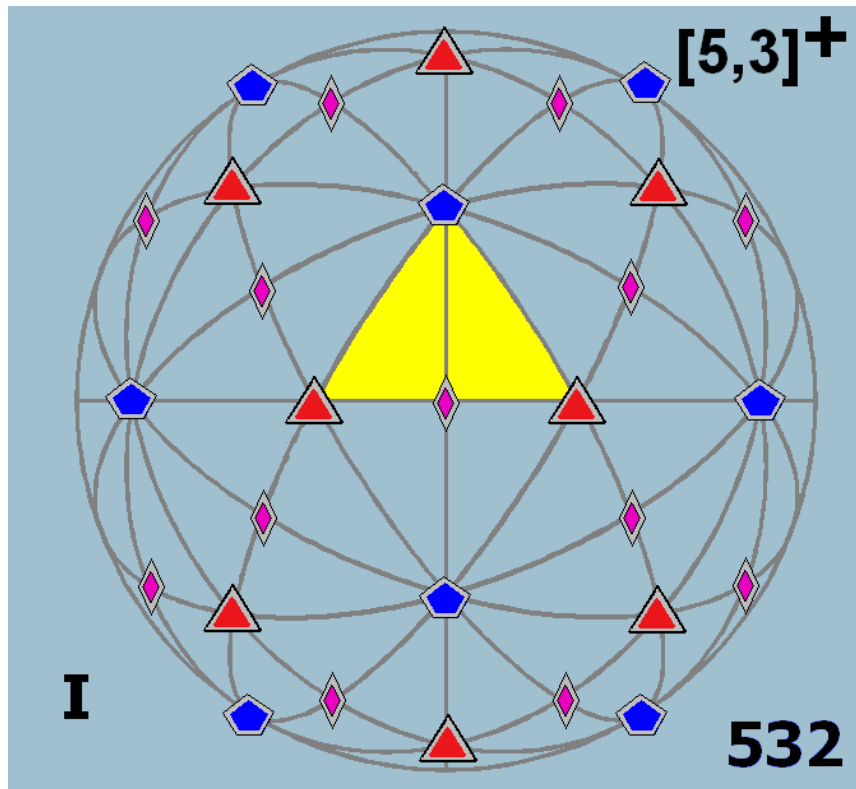
**Non-zero chirality** of fixed sign  $\varepsilon_i$  ;

$$\sum_i \varepsilon_i \neq 0:$$

→ any distribution is also chiral

***Rotational*** symmetry of an icosahedron ***I*** : ***no inversion nor mirror planes***

# Icosahedral Rotational Group $I$



Möbius Tessellation of a sphere by the action of Icosahedral Symmetry Elements with the fundamental domain of the Group  $I$  (called in Virology – “Asymmetric Protein Unit”)

Group  $G = I$  consists of 60 elements  
 $|G| = 60$

Dimension of its regular orbit of the group action on point coordinates is also  $\dim [\text{Orb}_G] = 60$

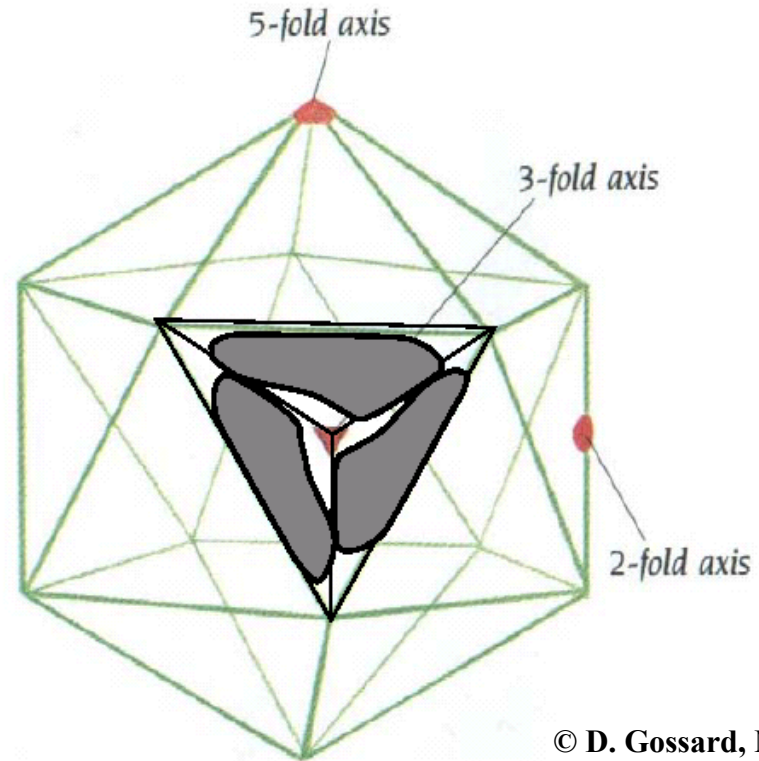
Maximal dimension for Irreducible 3D rotational group

To compare with  
 $\dim [\text{Orb}_G] = 24$  for  $G = O$   
octahedral rotational group  
and

$\dim [\text{Orb}_G] = 12$  for  $G = T$   
tetrahedral rotational group

# Icosahedral Symmetry

- 12 vertices
- 20 faces  
(equilateral triangles)
- 5-3-2 symmetry axes
- 60 identical\* subunits  
in identical environments  
can form icosahedral shell  
\* asymmetric  
and not 120 as it could be for  
symmetric subunits



© D. Gossard, MIT

## **Maximal number of identical asymmetric proteins in a shell**

To compare with 24 subunits in the octahedral shell, and 12 subunits in the tetrahedral shell cases, respectively.



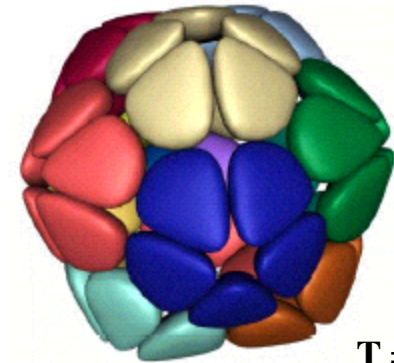
# Bigger Capsid shells

- Evolutionary pressure pushes to increase the genome size  
⇒ to make larger capsid ⇒ to use *more protein subunits*
- *Not possible* to form icosahedral shell of identical subunits *in identical environments* with more than 60 subunits
- Viruses with more than 60 subunits were observed
- *Questions* :
  - How can more than 60 subunits form an icosahedral shell ?
  - Will any number of subunits work ?
  - If so, how would they be organized ?

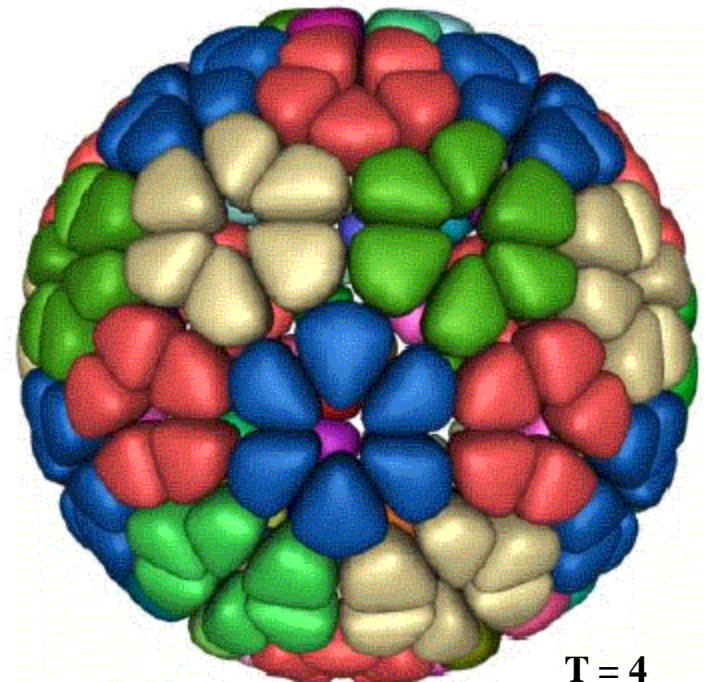
# Quasi-equivalence

D.L.D. Caspar & A. Klug, *Cold Spring Harbor Symp.* 27 (1962)

- Not all protein subunits are equivalent
  - *“Nearly-identical subunits” in “slightly different environments”*
- How to relate different environments ?
  - ⇒ Simple geometrical scheme ⇒
    - pentamers at vertices
    - hexamers elsewhere
- Only certain number of subunits can form an icosahedral shell
  - ⇒ Selection rules :
    - only  $N = 60 T$ , with  $T = h^2 + hk + k^2$  with  $h, k = 1, 2, 3, \dots$  can work



$T = 1$   
 $(h, k) = (1, 0)$



$T = 4$   
 $(h, k) = (2, 0)$

# Geometrical model of Quasi-equivalence

D.L.D. Caspar & A. Klug, *Cold Spring Harbor Symp.* 27 (1962)

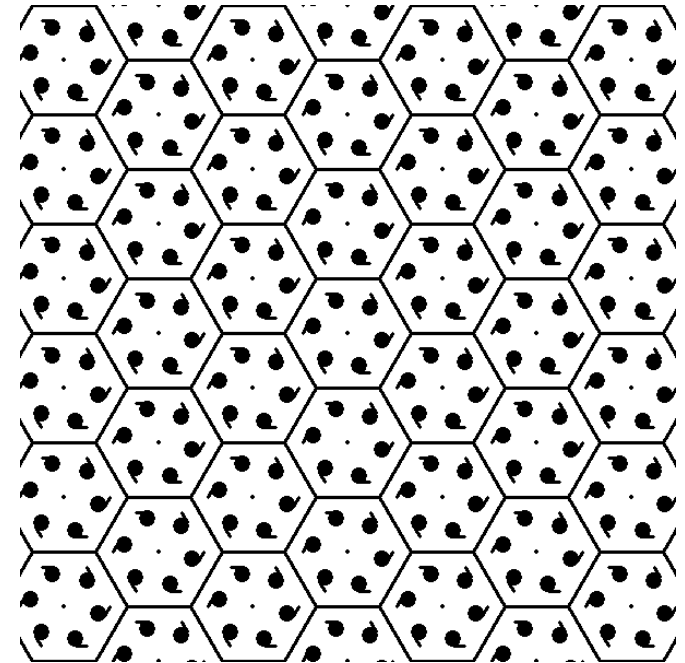
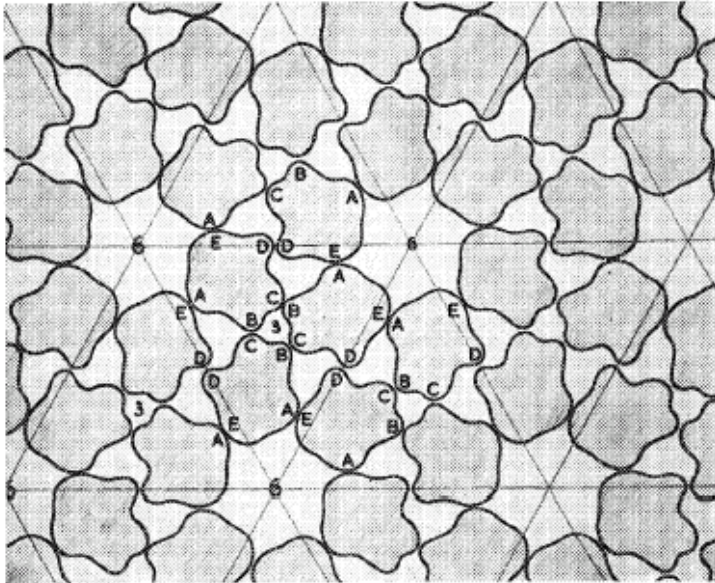


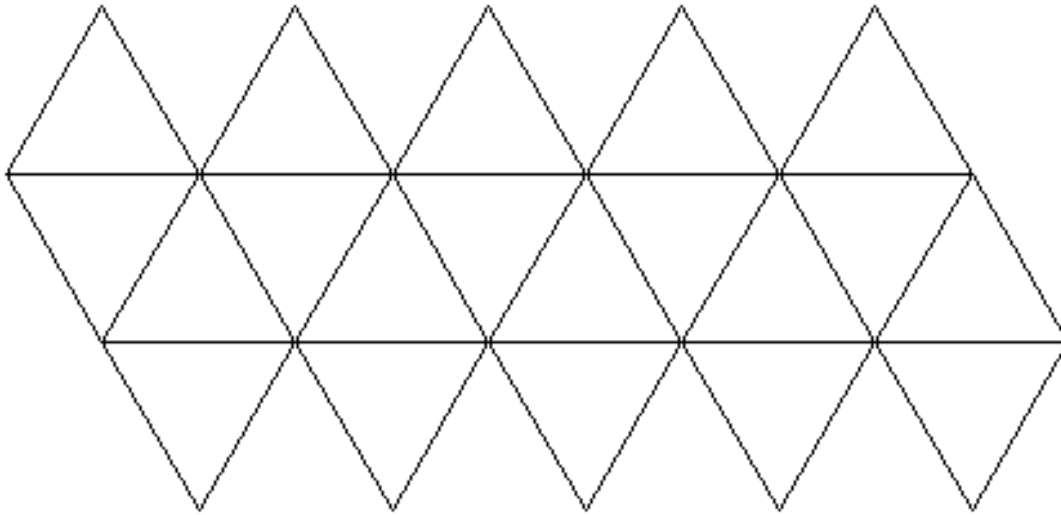
FIGURE 6. Asymmetric units arrayed in an equilateral-triangular plane net. Besides having translations, here  $a$  and  $a'$ , the lattice has 6-fold rotational axes of symmetry. Although the asymmetric units are in 6 different orientations in space, they are all exactly equivalently related.

Each unit here is equipped with five "bond" sites, A, B, C, D, and E, forming three different "bonds", namely a hexamer bond AE, a trimer bond BC, and a dimer bond DD. (Note that only two of these bonds are absolutely essential for coherence of the array.)

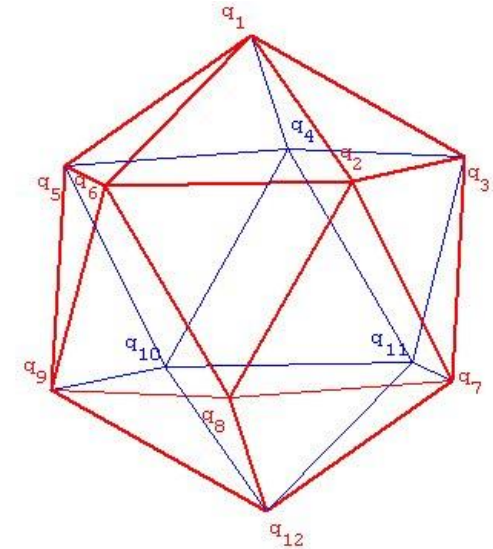
Asymmetric proteins in general positions in the hexagonal unit cell  
(regular orbit of the 2D space group)

Protein environments are equivalent due to the lattice translation

# Geometrical model of Quasi-equivalence



Net of an Icosahedron is **commensurate**  
with the hexagonal lattice  
Local order is hexagonal



Folded Icosahedron

Mapping of the Hexagonal Lattice to the Surface of an Icosahedron  
(“slitting the net and folding”)

# Geometrical model of Quasi-equivalence

**Selection rules :**

**Number T of different environments =  
Triangulation Number**

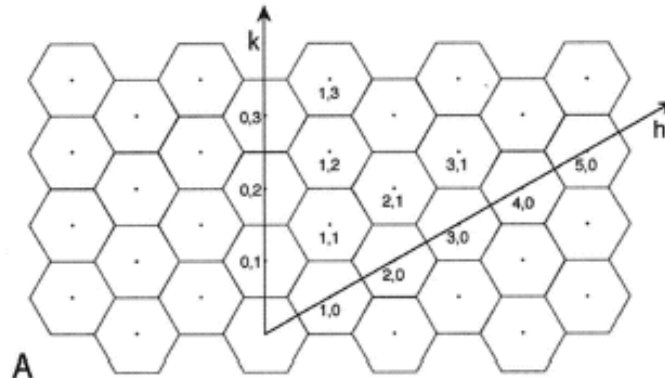
$$T = h^2 + hk + k^2$$

**But**

**Not unique**

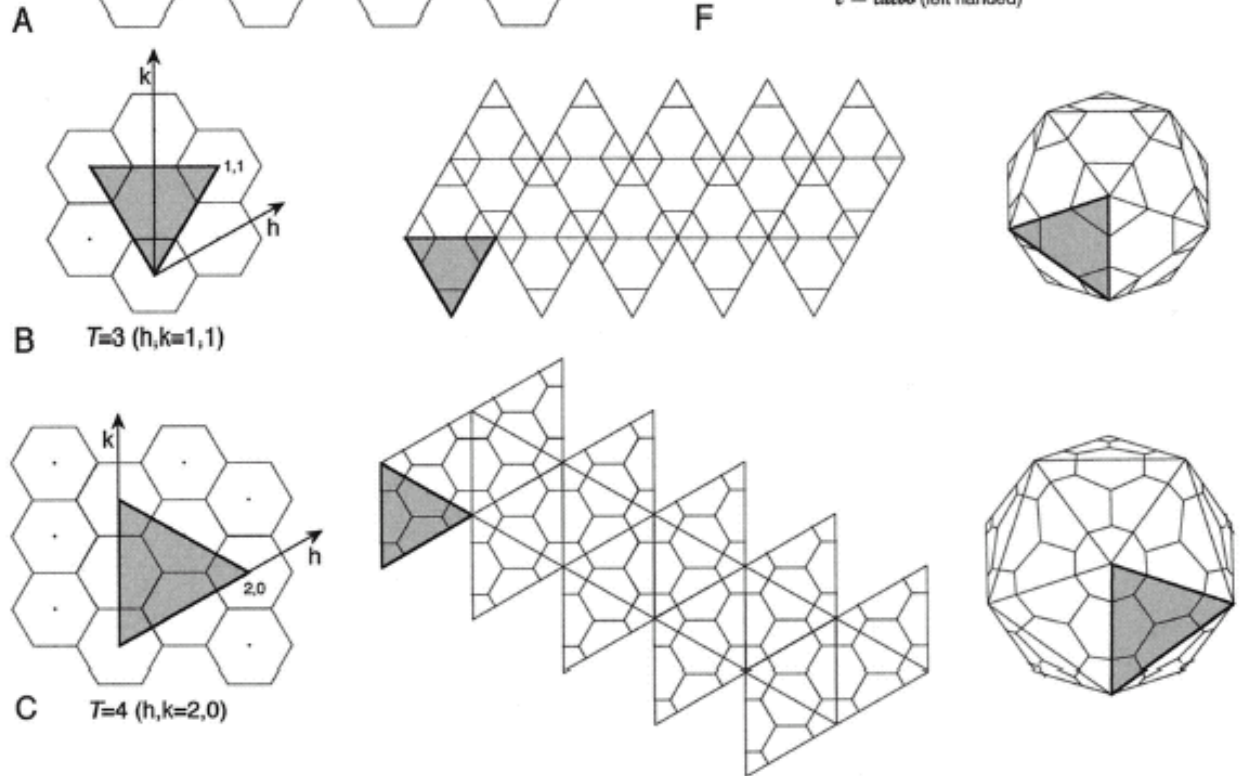
**+**

**No relation with  
assembly thermodynamics**



h	k	T	Example
1	0	1	bacteriophage $\phi$ X174
1	1	3	tomato bushy stunt virus
2	0	4	Sindbis virus
1	2	7 $\ell$	polyoma virus
3	1	13 $\ell$	reovirus
1	3	13 $\ell$	infectious bursal disease virus
4	0	16	herpesvirus
5	0	25	adenovirus

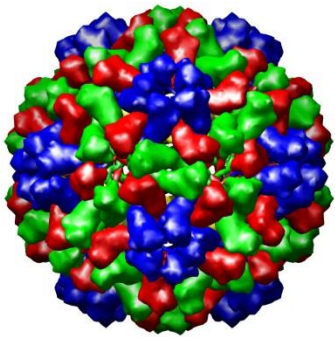
Notations:  $\ell$  = *letras* (right handed)  
 $\ell$  = *letras* (left handed)



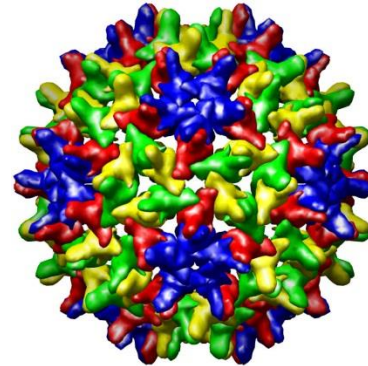


# Conventional and Unconventional Capsid Structures

Conventional « spherical » viral capsids exhibit spatial organization consistent with the Caspar & Klug selection rules

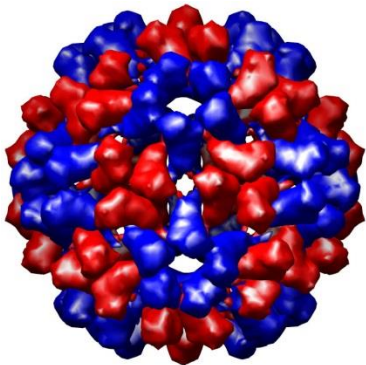


Cowpea Chlorotic  
Mottle Virus (CCMV)  
 $T = 3$

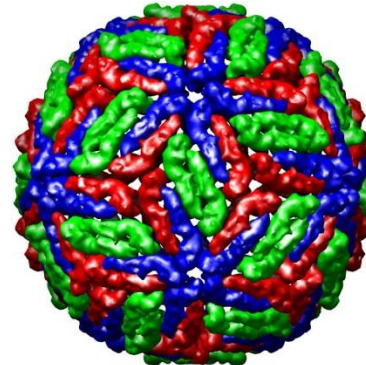


Hepatitis B Virus  
(HBV)  
 $T = 4$

**However, unconventional capsid structures don't**



L-A Virus  
 $T = 2$   
*forbidden by*  
Caspar-Klug  
*selection rules*



Dengue Virus  
 $T = 3$   
but *without*  
Caspar-Klug  
*hexamers*

**Looking for other ideas : Is it possible to propose common selection rules ?**

# Basic Ideas from Statistical Condensed Matter Physics

One type of proteins → One statistical *density distribution function*

Physical equivalence → Proteins located in a system of maxima of a *single irreducible* density function

Assembly thermodynamics → Free energy invariant wrt the density function

Simple analogs : Atomic solids

Physical equivalence in a simple system → Atoms in atomic crystals (lattice nodes)

Physical equivalence in a more complex system → Atoms in atomic quasicrystals

Thermodynamics → Crystallization theory : Density waves

# Landau Theory of Crystallization : Static Density Waves

$\rho = \rho_0 + \Delta \rho$   
Density in the  
crystal state

Classical crystal case:  $\Delta \rho = \sum_{\mathbf{k}} \rho_{\mathbf{k}} \exp(i\mathbf{k}\mathbf{r})$

**Density** deviation from its value  $\rho_0$  in the isotropic state =  
= **System of Plane Waves with the fixed length of wave vectors  $|k|$**

**Critical System of Density Waves**  $\Delta \rho = \sum_{\mathbf{k}_i} \rho_{\mathbf{k}_i} \exp(i\mathbf{k}_i\mathbf{r})$   
where  $\rho_{\mathbf{k}_i}$  are Order Parameter Components

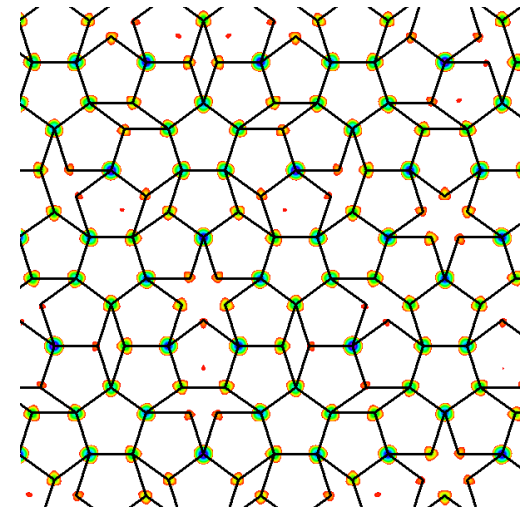
**For crystals formed by one type of atoms the atomic positions are associated with the maxima of the Critical System of Density Waves (CSDW)**

**Quasicrystals:**

**Small finite number of different environments  
generated by a single irreducible density function**

Per Bak, Phys. Rev. Lett. **54**, 1517 (1985)

S.B. Rochal et al. , Phys. Rev. B. **72**, 024210 (2005)

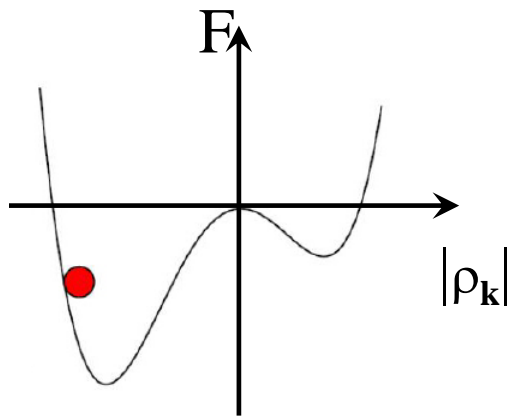


# Landau Theory of Crystallization : Static Density Waves

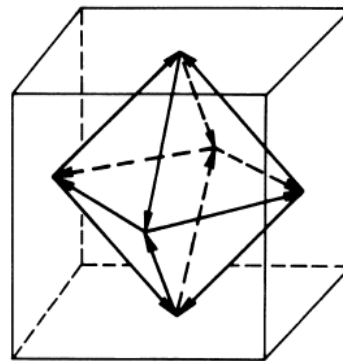
Free energy expansion near the isotropic-to-crystal phase transition :  $F_0 + F_2 + F_3 + F_4 + \dots$

$$F_2 = \int d^2k A_{\mathbf{k}} \rho_{\mathbf{k}} \rho_{-\mathbf{k}} = A(T, c) |\rho_{\mathbf{k}}|^2 ; \quad F_3 = B(T, c) \sum_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3} \rho_{\mathbf{k}_1} \rho_{\mathbf{k}_2} \rho_{\mathbf{k}_3} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

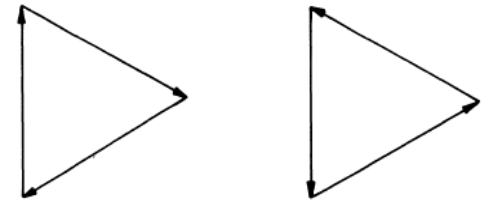
$$F_4 = C(T, c) \sum_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4} \rho_{\mathbf{k}_1} \rho_{\mathbf{k}_2} \rho_{\mathbf{k}_3} \rho_{\mathbf{k}_4} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)$$



*B.C.C. Lattice*



*2D Hexagonal Lattice*



S. Alexander & J. McTague,  
Phys. Rev. Lett. **41**, 702 (1978)

Currently :  
In P. Chaikin & T. Lubensky Textbook  
« Principles of Condensed Matter Physics »

*Contributions to the Cubic Term in  
the Free Energy – Choice of the  
thermodynamically favorable state*

# Assembly of protein shells : Spherical Density Waves

$\rho = \rho_0 + \Delta \rho$   
Density in the  
self-assembled  
state

2D spherical distribution of finite asymmetric units in 3D space:

$$\Delta \rho = \sum_{l \in \mathbb{N}} \sum_{|m| \leq l} \rho_{lm} Y_m^l(\Theta, \phi)$$

**Density** deviation from its value  $\rho_0$  in the isotropic state =  
= ***System of Waves on a Sphere with the fixed wave number  $l$***

***Asymmetric Proteins*** have no Proper Symmetry. Because of the Asymmetry the final structure has ***neither spatial inversion nor symmetry plane*** elements => ***only odd spherical harmonics*** in  $\Delta \rho$

Restrictions on the free energy form:

**Free energy density expansion** near the assembly transition :  $F_0 + F_2 + F_3 + F_4 + \dots$

$$F_2 = A(T, c) \sum_m a_m \rho_{lm} \rho_{l(-m)}$$

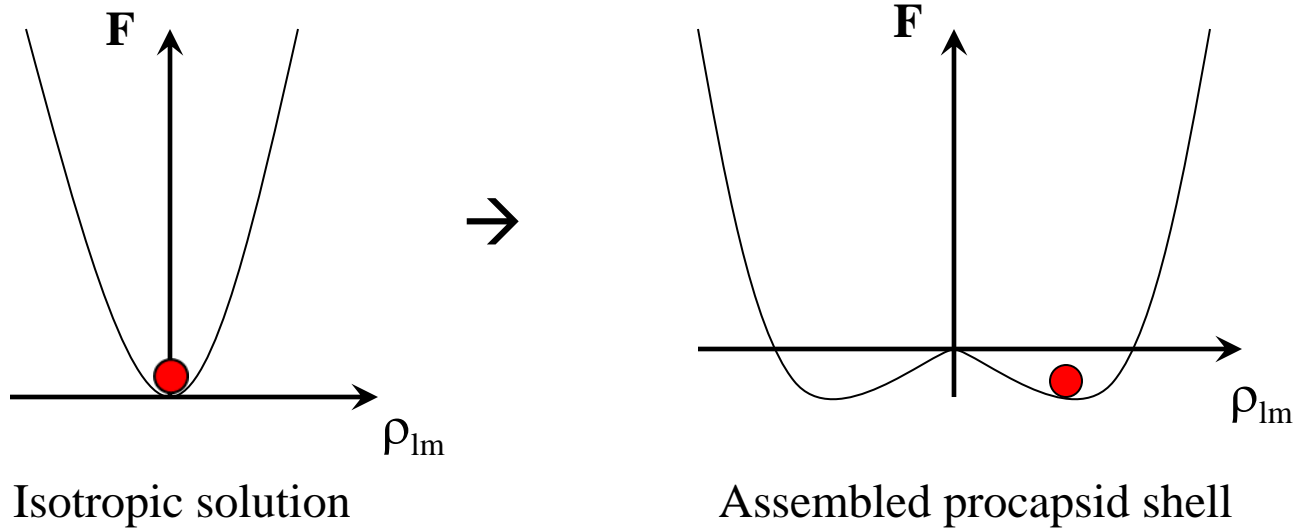
$$F_3 = B(T, c) \sum_{m_1, m_2, m_3} a_{m_1, m_2, m_3} \rho_{lm_1} \rho_{lm_2} \rho_{lm_3} \delta(m_1 + m_2 + m_3) \equiv 0 \quad (!!!)$$

$$F_4 = \sum_k C_k(T, c) \sum_{m_1, m_2, m_3, m_4} a_{m_1, m_2, m_3, m_4} \rho_{lm_1} \rho_{lm_2} \rho_{lm_3} \rho_{lm_4} \delta(m_1 + m_2 + m_3 + m_4)$$

Possibility of 2<sup>nd</sup> order transition → ***specific kinetics***



# Assembly of protein shells : Spherical Density Waves



**No coexistence of two states** → **Rare intermediate products of assembly**  
→ **Slow dynamics**

→ **No one-by-one steps** → **No polymerization-like assembly process**

Experiment:

- **no incomplete capsids**, either isolated proteins in solution, or fully assembled shell;
- very slow dynamics called **“protein sitting”**

# “Structural” selection rules

**What is proposed instead of Caspar-Klug selection rules for T number ?**

→ **Symmetry restriction on the choice of density functions :**

Density function invariant with respect to the rotational icosahedral symmetry group  $I$  can be constructed *not for all odd  $l$*

but only for  $l = 15 + 6i + 10j$  ;  $i, j = 0, 1, 2, 3, \dots$

Protein distribution in capsids of small viruses → icosahedral density waves with  $l = 15, l = 21, l = 25, l = 27, l = 31$ , etc.

→ **Restriction on possible protein positions**

However , **no limitation of the Caspar-Klug type** : all integer T are possible

Number of different types of maxima = Number T of protein environments

→ for a **small viruses** (with  $l < 44$ ): Icosahedral density functions

$$\Delta \rho_l(\Theta, \phi) = \sum_{|m| \leq l} \rho_{lm} Y_m^l(\Theta, \phi) = B f_l(\Theta, \phi)$$

are *unique functions without fitting parameters*

# Selection rules

Critical System of Density Waves (CSDW) in the considered case :

$$\Delta \rho_{cr} = B f_l(\Theta, \phi) = \sum_{|m| \leq l} \rho_{lm} Y_m^l(\Theta, \phi)$$

where  $\rho_{lm}$  span active irreducible representation (IR) of the symmetry group  $G_0$  of the parent state. « Active IR drives the transition ».

Active IR of  $G_0$  must subduce the identity representation of the symmetry group  $G$  of the ordered state ( $G \subset G_0$ ).

$B f_l(\Theta, \phi) = \sum_{|m| \leq l} \rho_{lm} Y_m^l(\Theta, \phi)$  must span the identity representation of  $G$ .

**The representation subduced from  $G_0$  to  $G$  must contain the identity representation of  $G$ .**

Frequency of subduction:  $n_l = (I/|G|) \sum_G \chi(g)$

$\chi(g)$  is the character of the  $G_0 = SO(3)$  group element  $g$ ;  
the sum runs over the elements  $g \in G$  of the icosahedron rotation group  $I$  ;  
 $|G|=60$  is the  $I$  group order

Subduction Criterion:  **$n_l \neq 0$  or  $n_l = 0 \rightarrow$  selection rules for  $l$ :**

# Selection rules

## Constructive form

Frequency of subduction:  $n_l = (I/G) \sum_G \chi(g)$

$\chi(g)$  is the character of the element  $g \in \text{SO}(3)$  :  $\chi(l, \alpha) = \frac{\sin[(l + 1/2)\alpha]}{\sin(\alpha/2)}$

Conjugacy classes of the  $I$  group:

- Identity E
- 15 rotations  $C_2$ , order 2
- 20 rotations  $C_3$ , order 3
- 12 rotations  $C_5$ , order 5
- 12 rotations  $(C_5)^2$ , order 5

$$n(l) = (1/60) [2l + 1 + 15\chi(l, \pi) + 20 \chi(l, 2\pi/3) + 12 \chi(l, 2\pi/5) + 12 \chi(l, 4\pi/5)]$$

$$\mathbf{n_l \neq 0 \text{ for } l = 15 + 6i + 10j ; \quad i, j = 0, 1, 2, 3, \dots}$$

# Selection rules and groups not generated by reflections

CSDW  $B f_l(\Theta, \phi)$  are homogeneous functions of degree  $l$

Any scalar function invariant wrt  $I$  group :  $F(g \vec{r}) = F(\vec{r}) \quad g \in I, \vec{r} = (x,y,z)$

can be expanded in formal series  $F(J_0, J_1, J_2, J_3)$

$\{J_i\}$  is the **integrity basis** –

full set of generators of the ring of polynomials invariant wrt  $I$  group:

$$J_0 = x^2 + y^2 + z^2 \quad J_1 = \prod_{i=1}^6 \vec{n}_i \cdot \vec{r} \quad J_2 = \prod_{i=1}^{10} \vec{p}_i \cdot \vec{r} \quad J_3 = \prod_{i=1}^{15} \vec{q}_i \cdot \vec{r}$$

$I$  group is **not generated by reflections**  $\rightarrow (J_0, J_1, J_2, J_3)$  form **syzygy** -  
an algebraic relation of the form  $(J_3)^2 = P(J_0, J_1, J_2)$  in 30th degree

$\rightarrow$  Invariant homogeneous functions of degree  $l$

$$G_l(x,y,z) = J_3 \left[ \sum_{15+2k+6i+10j=l} A_{k,i,j} (J_0)^k (J_1)^i (J_2)^j \right]$$

on a sphere  $J_0 = \text{Const}$   $\rightarrow$

$$B f_l(\Theta, \phi) = J_3 \left[ A_{0,0} + A_{1,0} J_1 + A_{0,1} J_2 + A_{2,0} (J_1)^2 + A_{1,1} J_1 J_2 + \dots + \sum_{15+6i+10j=l} A_{i,j} (J_1)^i (J_2)^j \right]$$

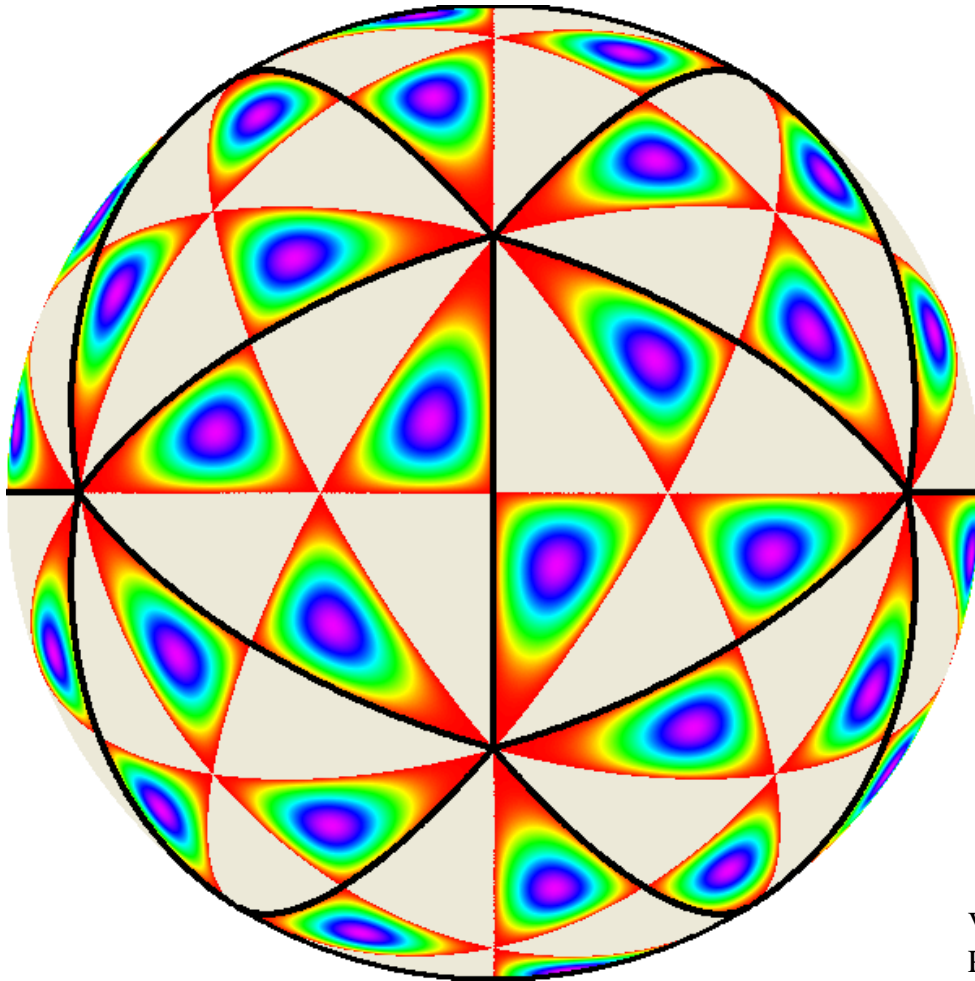
$$\rightarrow \mathbf{l = 15 + 6i + 10j}; \quad i, j = 0, 1, 2, 3, \dots$$



# Irreducible icosahedral density functions

The explicit form is obtained by averaging of  $Y_m^l(\Theta, \phi)$  over the  $I$  symmetry group

$$\Delta \rho_l(\Theta, \phi) \propto f_l(\Theta, \phi) = (1/60) \sum_G Y_m^l(\Theta, \phi)$$



**Protein density distribution with  
the minimal possible wave  
number :**

$$l = 15$$

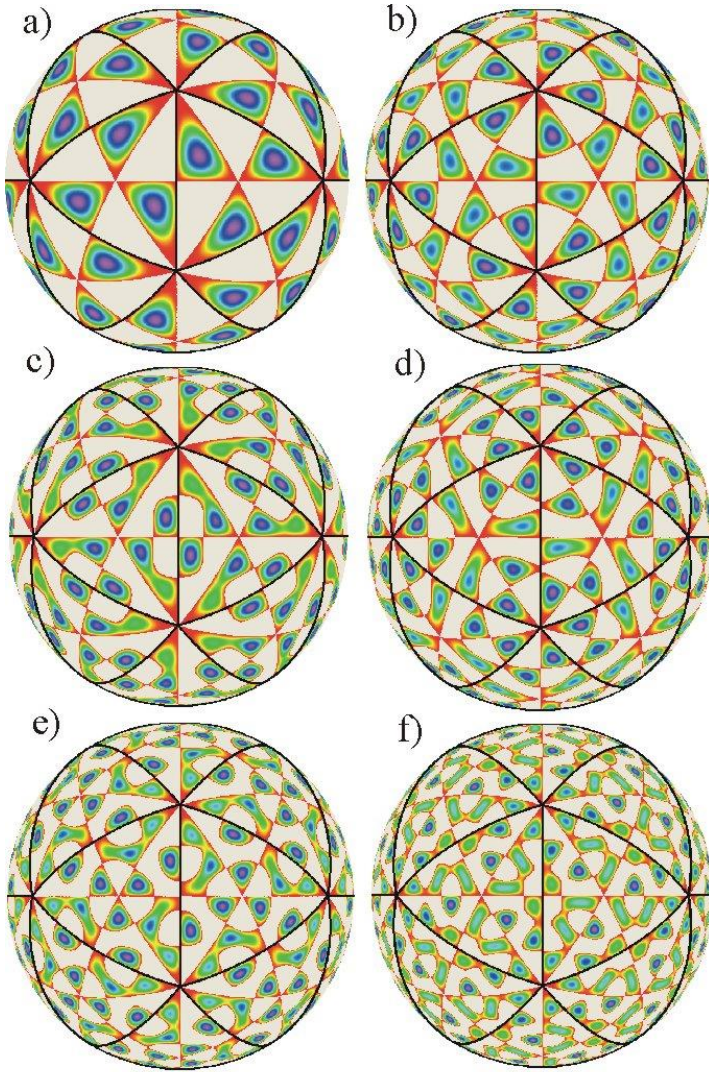
**60 equivalent density maxima in  
equivalent environments**

**T=1 capsids**

V.L. Lorman & S.B. Rochal , Phys. Rev. Lett., (2007),  
Phys. Rev. B (2008), Phys. Rev. E (2009), Phys.Rev. Lett. (2012)

Research Highlights : Nature Nanotechnology

# Classification of protein density functions



Density functions for several small icosahedral viruses

a)  $l = 15$ ;  $T = 1$  (Caspar-Klug structure)

b)  $l = 21$ ;  $T = 2$  (**non Caspar-Klug structure**)

c)  $l = 25$ ;  $T = 3$  (**non Caspar-Klug structure**)

d)  $l = 27$ ;  $T = 3$  (Caspar-Klug structure)

e)  $l = 31$ ;  $T = 4$  (Caspar-Klug structure)

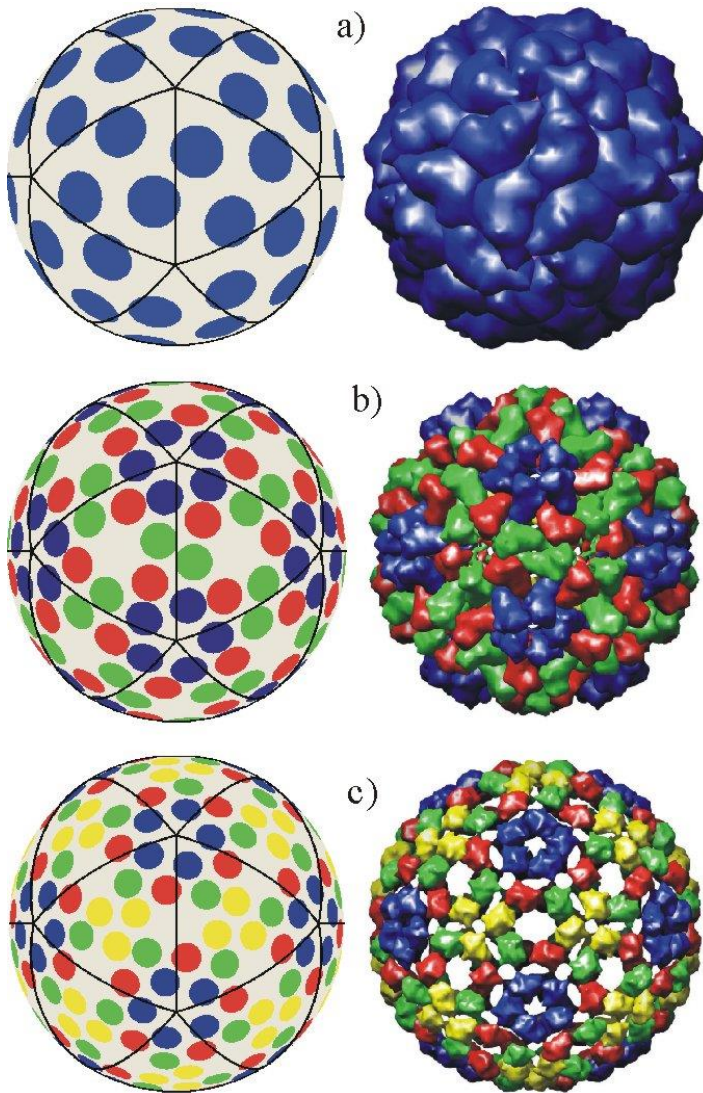
f)  $l = 37$ ;  $T = 6$  (**non Caspar-Klug structure**)

Viruses can have the **same T** number but **qualitatively different organization** (see c and d)

Figure 1

# Predicted protein distributions and viral structures

## Examples of viruses which satisfy Caspar-Klug rules



a)  $l = 15$ ;  $T = 1 \rightarrow$  Satellite Tobacco Necrosis Virus

b)  $l = 27$ ;  $T = 3 \rightarrow$  Cowpea Chlorotic Mottle Virus

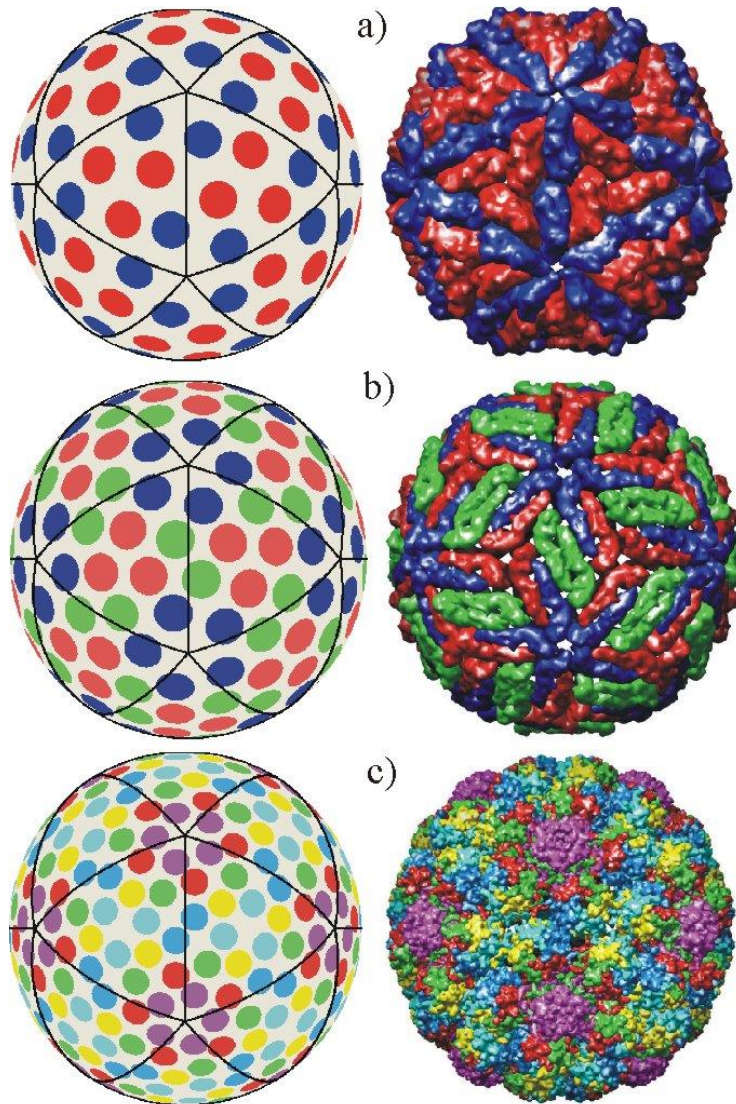
c)  $l = 31$ ;  $T = 4 \rightarrow$  Semliki Forest Virus

*Small finite number of different environments generated by a single irreducible density function*

Figure 2



# Predicted protein distributions and viral structures



## Examples of viruses which do not satisfy Caspar-Klug rules

a)  $l = 21$ ;  $T = 2 \rightarrow$  L-A Virus

$$T \neq h^2 + hk + k^2$$

b)  $l = 25$ ;  $T = 3 \rightarrow$  Dengue Virus

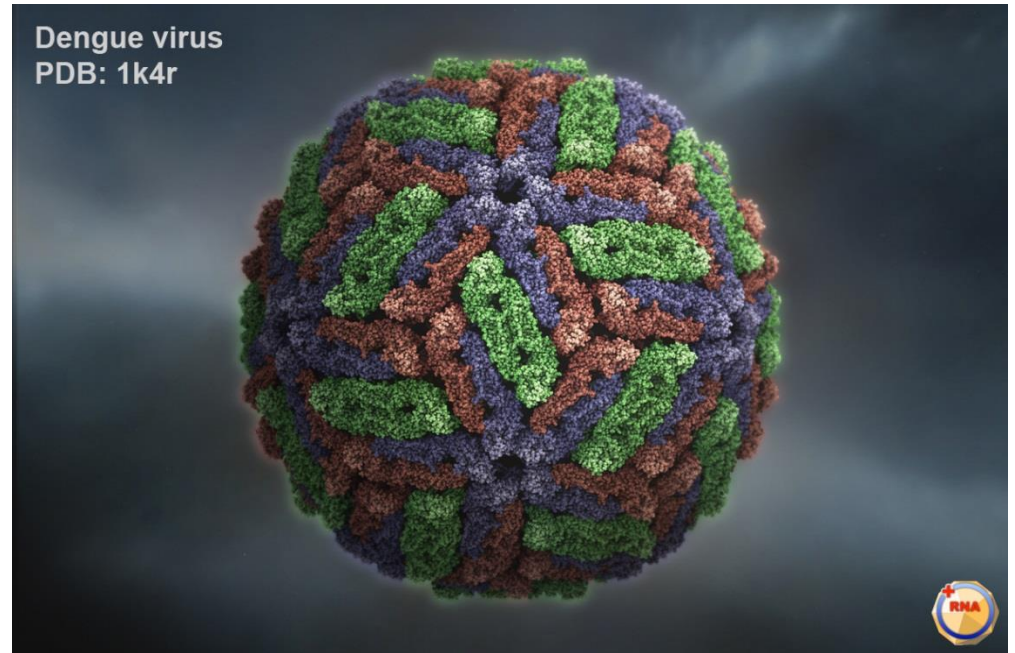
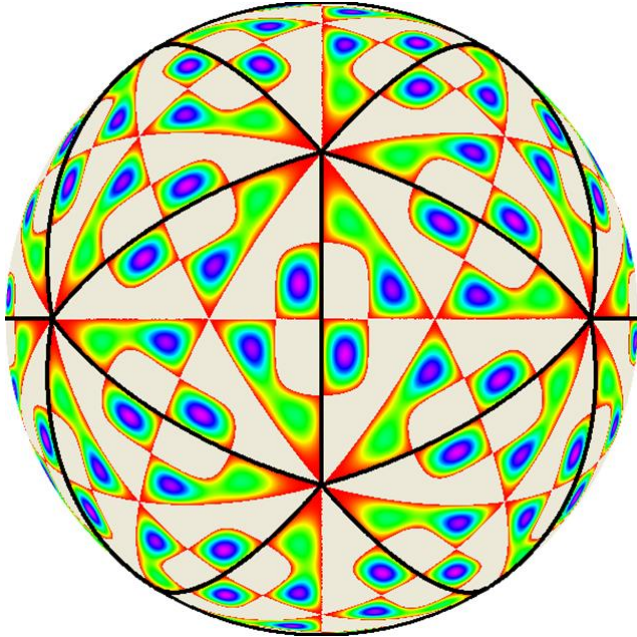
no hexamers

c)  $l = 37$ ;  $T = 6 \rightarrow$  Murine Polyoma Virus

$$T \neq h^2 + hk + k^2$$

Figure 3

# Dengue Virus Capsid



T=3 environments; N= 180 proteins  
**Typical rhombic motif without hexamers**

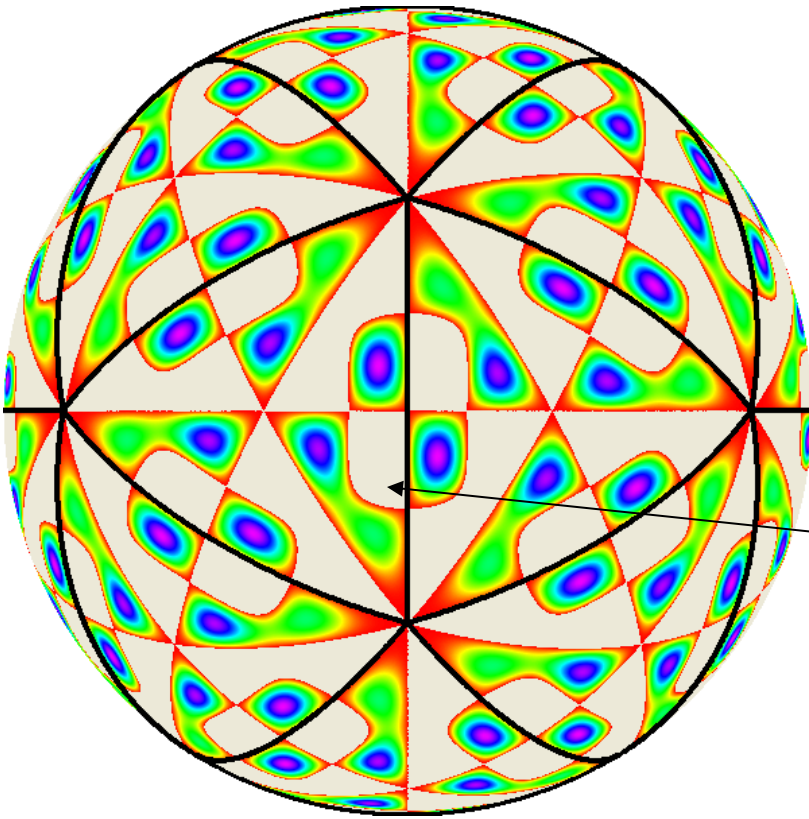
Icosahedral density distribution function with  $l=25$

Capsid “breathing” effect

K.A. Dowd, et al., J. Virol **88**, 11726 (2014), NIH & Purdue teams

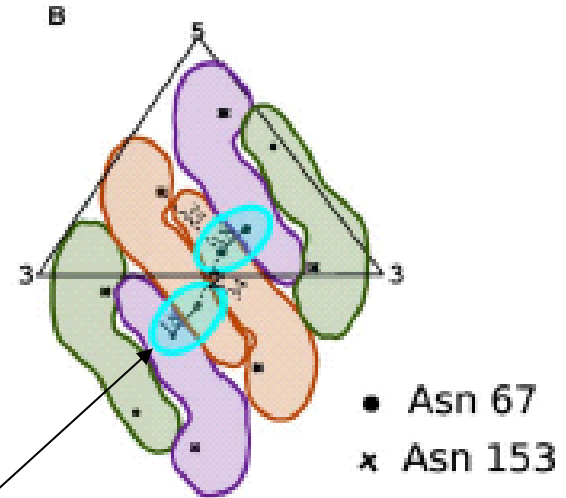


# Infectivity and structure

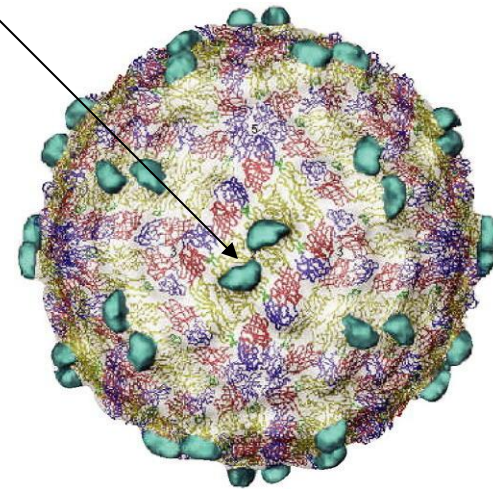


Cell receptor carbohydrate recognition domains are bound to the **Dengue Virus** in the *deepest minima* of the icosahedral density distribution function with  $l = 25$ , *near its highest maxima*

⇒ **Relation with binding probability**

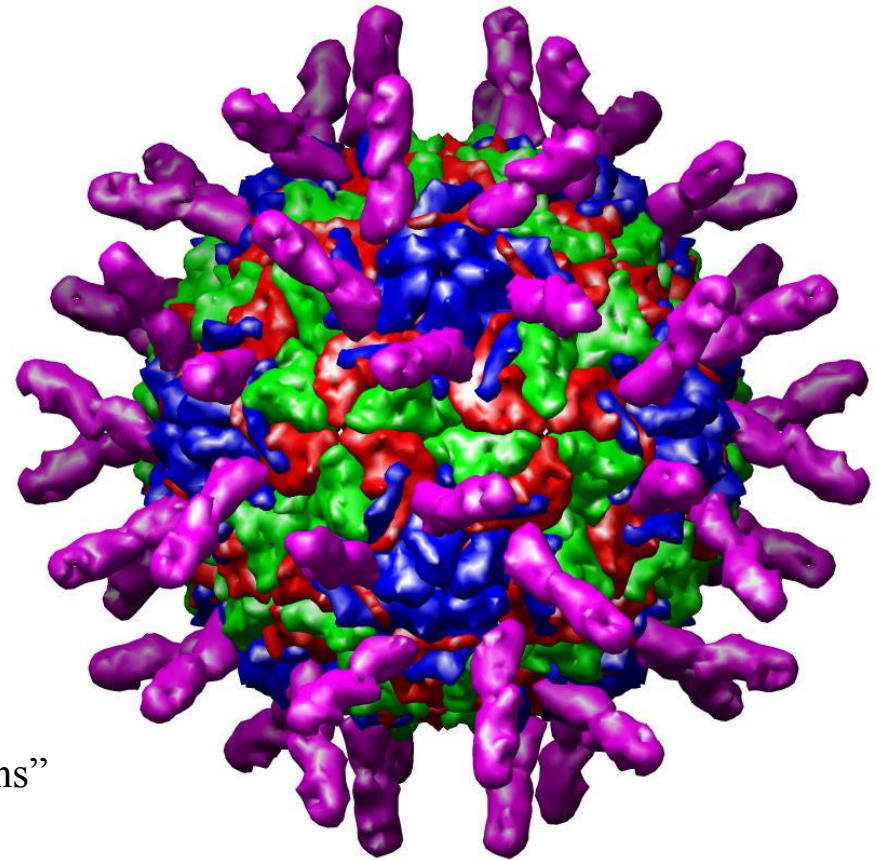
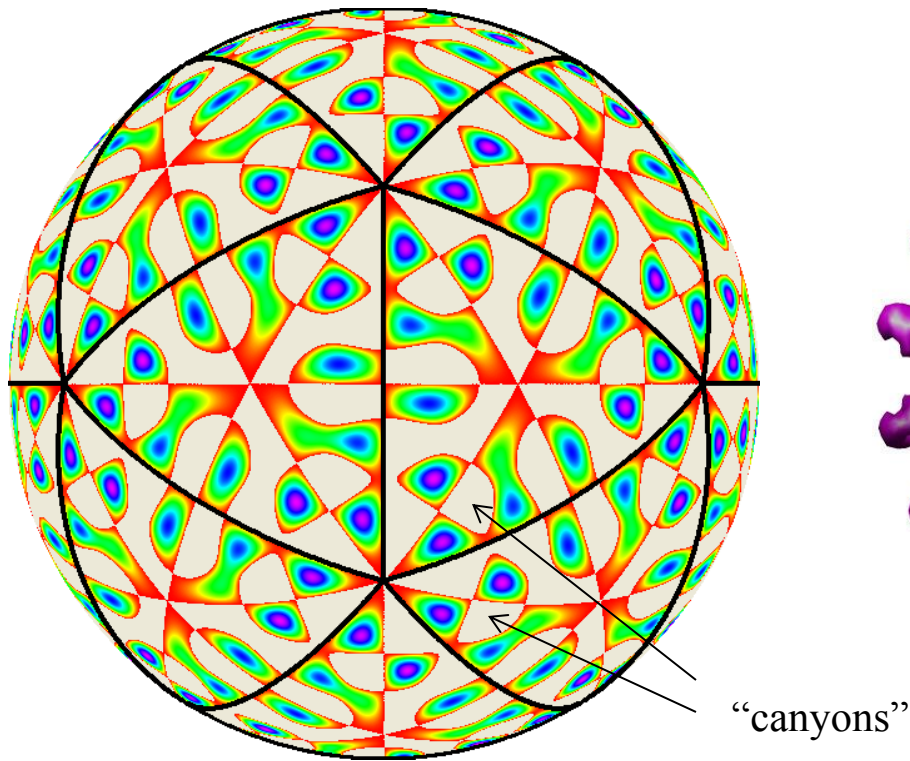


Cell receptor domains bound to the **Dengue Virus** surface



E. Pokidysheva et al. , Cell **124**, 485 (2006)  
M. Rossmann's group, Purdue University

# Infectivity and structure

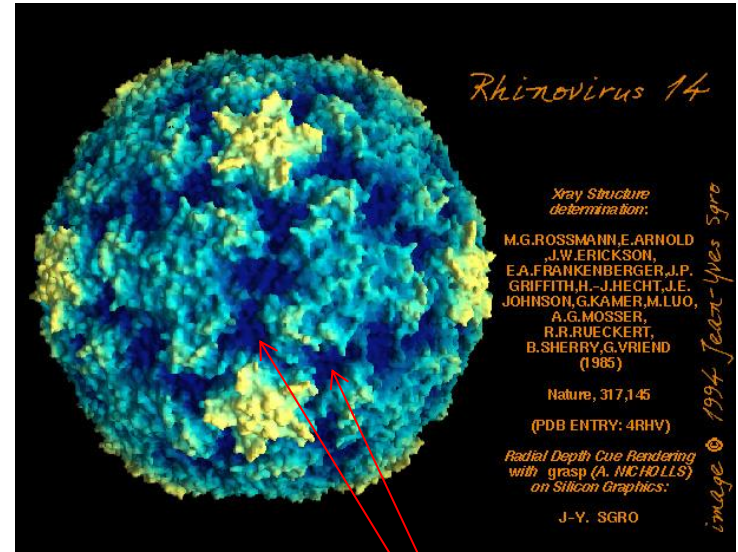
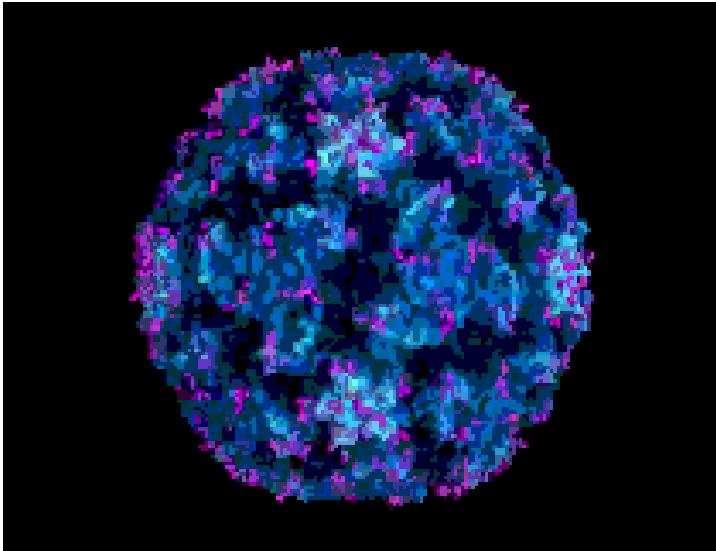


ICAM-1 receptor domains are bound to the Human Rhinovirus Virus in the deepest minima of the icosahedral density distribution function with  $l = 33$  forming narrow “canyons”

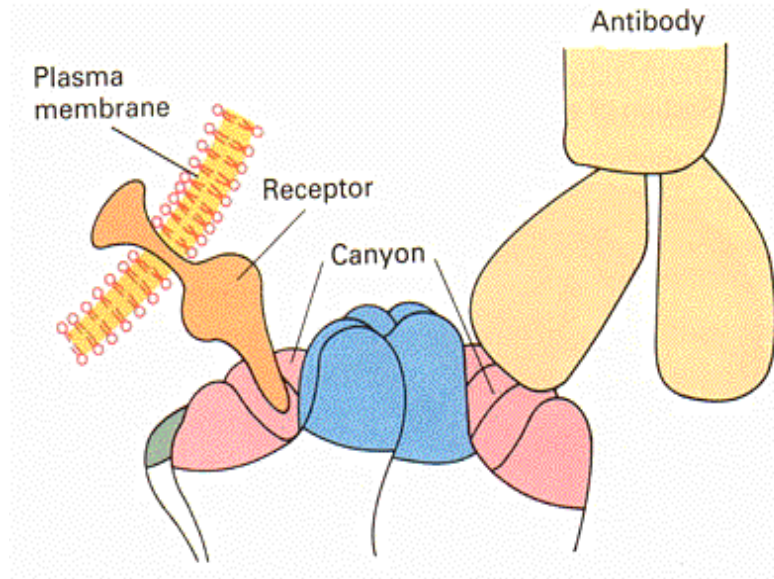
**Human Rhinovirus** with the fragment of its cellular receptor ICAM-1 (intracellular adhesion molecule – 1)  
ICAM-1 binds to the HRV « canyon »

**=> Relation with binding probability**

# Infectivity and structure



“canyons”



ICAM-1 binds to the HRV « canyon » too narrow for antibodies

M. Rossmann

# Intermediate Summary

- Asymmetric protein assembly in capsids of small icosahedral viruses :
  - Generalization of the Caspar & Klug theory of quasi-equivalence
  - Generalization of the Landau theory of crystallization
- Selection rules and method for protein density distribution function construction
  - Rules for protein distribution beyond the Caspar & Klug scheme
  - Relation between protein density distribution and binding probability

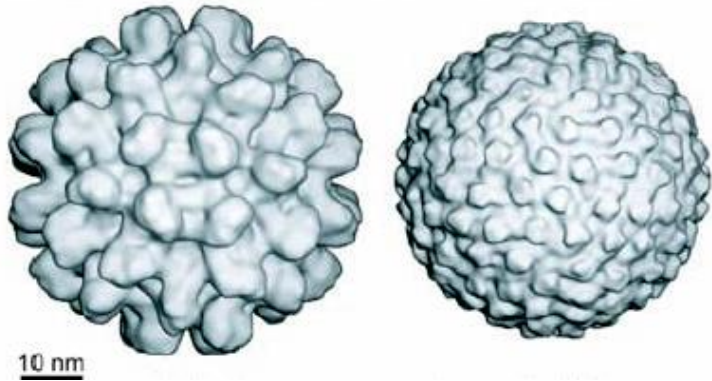
*in the same frame :*

- Protein rearrangement across reversible procapsid-capsid transitions during maturation in certain viruses
- Capsid polymorphism in viruses, mutants and virus-like particles
  - Different icosahedral shells formed by the same protein:  
Relevant density-wave parameters
- Non-icosahedral shells. Vault virus-like nanoparticles



Fundamental quantity conserved across the transition :  
Average wave vector of the density wave

Plane waves approximating density waves on a sphere : average wave vector  $q = l/R$   
related to a typical protein size



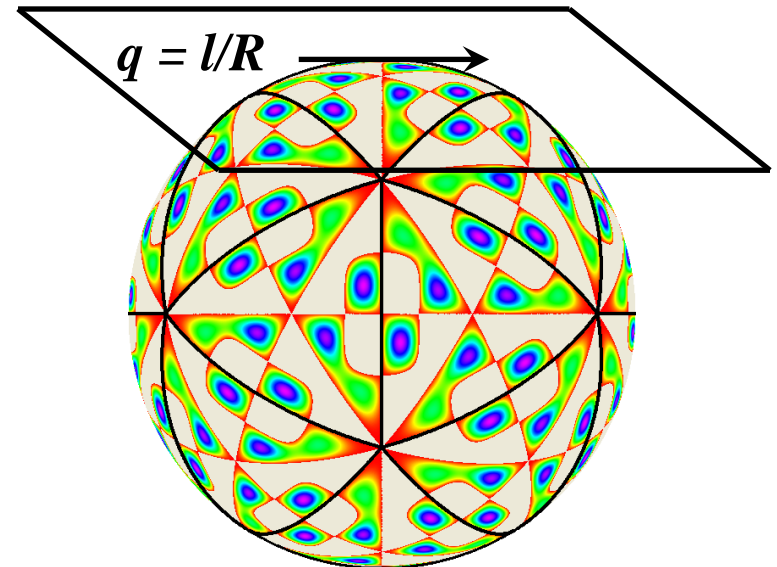
$l = 27; 2R = 59 \text{ nm}$      $l = 25; 2R = 53 \text{ nm}$

Dengue Virus

Strong size variation :  $R_{\text{procapsid}} / R_{\text{capsid}} \approx 13\%$   
but  $(l/R)_{\text{procapsid}} \approx (l/R)_{\text{capsid}}$

Approximating plane wave :  $\Delta \rho \sim \exp(i\mathbf{q}\mathbf{r})$

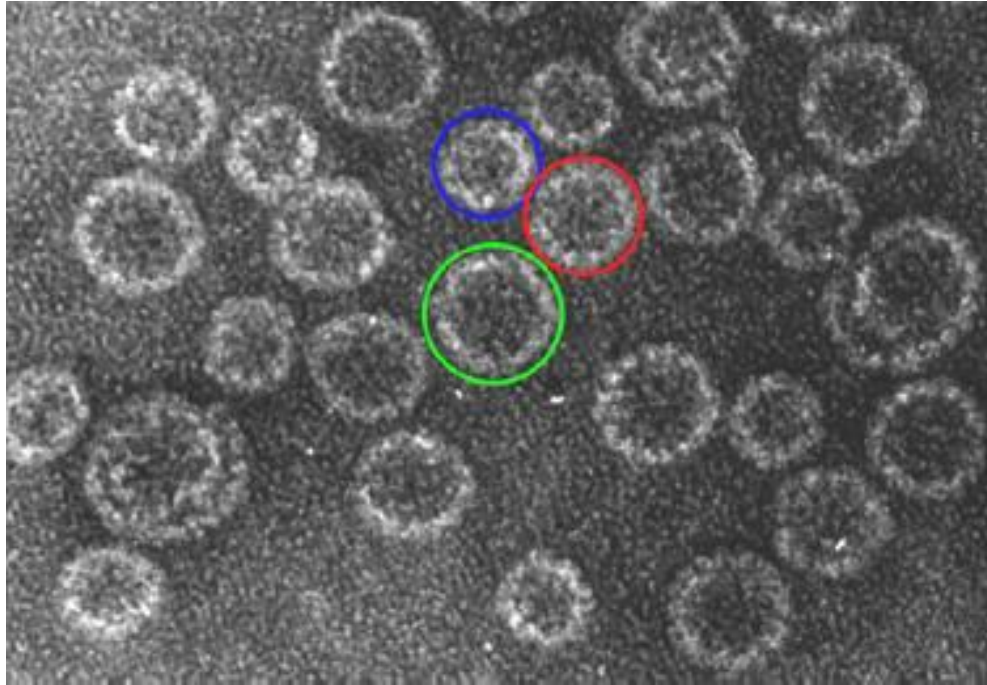
To compare with the SAXS on the isotropic protein solution before the assembly. Protein “atomic factor”.



# Capsid polymorphism : Same proteins forming different capsids (with different number of environments) :

Example:  
mutant Cowpea  
Chlorotic Mottle  
Virus (CCMV)

J.H. Tang et al.,  
J. Struct. Biol. **154**, 59 (2006)



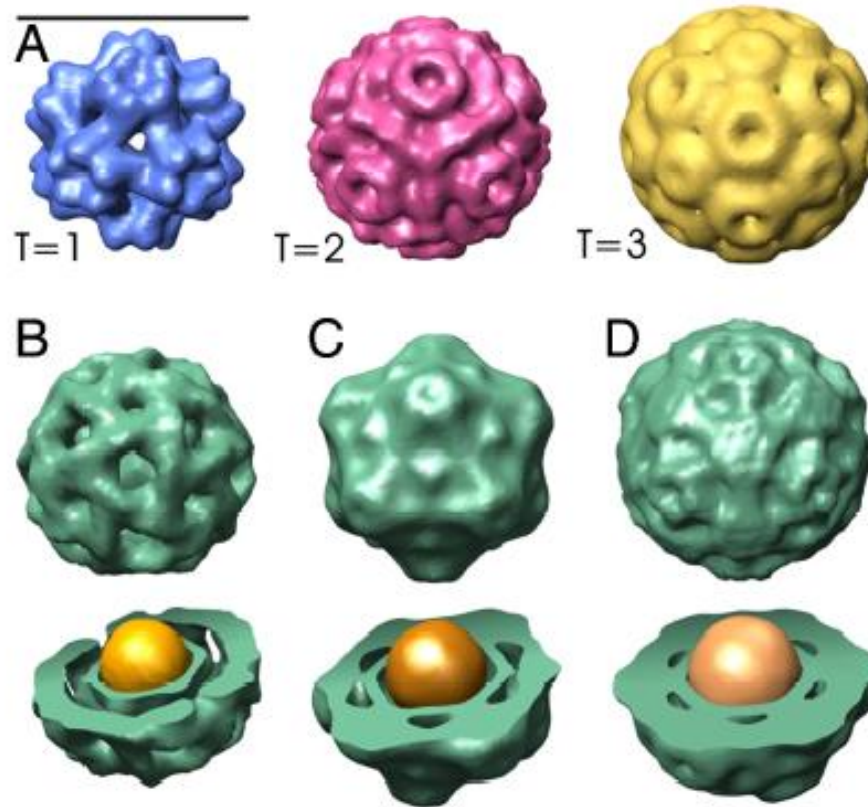
Mutant Capsid Proteins lacking most of the N-terminal domain, ND34

Blue, red, and green circles show the three different types of particles corresponding to the  $T = 1$ ,  $T = 2$ , and  $T = 3$  capsids, respectively.

Capsid type is “flexible” and may readily adapt to new requirements as the virus evolves



# Virus-Like Particles using Capsid polymorphism



J. Sun et al., PNAS **104**, 1354 (2007)

Core = gold nanoparticle functionalized with the carboxylated PEG; Coat = BMV Capsid Proteins  
Three different (but  $\approx$  appropriate) nanoparticle sizes  $\rightarrow$   
Three different VLP with coats  $\approx$  T = 1; T = 2, and T = 3

Drug delivery optimization



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