

# Physics as a (unique) nonlinear math which is free of infinities in solutions

Ivan L. Zhogin

*Bourestnik, Inc., Saint Petersburg*

zhogin@mail.ru

## Abstract

The fundamental physical theory should be nonlinear (to describe interactions) but that leads to problems, infinities in solutions – both for classical and quantum theories (not so toy, able to include gravity). String theory changes the very topology of Feynman diagrams, but still remains just a program, a design on a (complete) theory.

On the other hand, Einstein's research did not stop on Special and General Relativities – he also considered (compatible, second order) field equations whose symmetry group united symmetries of both relativity theories. Among such equations (mostly non-Lagrangian) one can find a unique one (no free parameters,  $D=5$ ) which solutions last forever (no breaks on singularities). Many features of this equation and its solutions are interesting indeed (three linearly unstable polarizations, plus longitudinal one, which add nothing to D-momentum; topological charges and quasi-charges) and give rise to a new interpretation of Quantum Mechanics and new 4th-order Lagrangian gravity.

## 1 Introduction

Social laws vary in space-time (Hammurapi's code, Rome law, Jackson-Vanik amendment); sometimes they would be ignored. Physical laws, on the contrary, are seemingly invariable, and hardly ignorable; supposedly, they should be united in a single law, the fundamental theory (FT).

Einstein believed that our world is simple (otherwise “is not interesting”), and understanding of a tiny part of the universe is all we need – other parts are similar! But how on earth can that tiny and simple part hold information on all those “fundamental constants”?

Perhaps, FT should have no arbitrary parameters (being perfect and beautiful indeed) – otherwise it's a huge set of similar theories, not a single theory at all. “Constants” might emerge as slowly varying and globally extended parameters of solutions, not of the theory itself (if simple, FT should be absolutely comprehensible).

## 2 “Unification” of Special and General Relativities

The general relativity theory (GRT) and the Standard Model of elementary particles engage respectively coordinate diffeomorphisms and Lorentz transformations as their (universal) symmetry

groups. The last (global group) fairly defines the space-time signature (a global feature), while the former breaks absolutism of Minkowski spacetime (with inertial coordinates), and leads to geodesics and the equivalence principle. Absolute parallelism (AP) embraces both symmetries. AP's (co)frame field  $h^a_\mu$  holds one Greek and one Latin index, the necessary minimum, and field equations are symmetrical under the following transformations [1,2]:

$$\tilde{h}^a_\mu(\tilde{x}) = \kappa \sigma^a_b h^b_\nu(x) \frac{\partial x^\nu}{\partial \tilde{x}^\mu}; \quad \kappa > 0, \quad \sigma^a_b \in O(1, D-1), \quad \det\left(\frac{\partial x^\nu}{\partial \tilde{x}^\mu}\right) \neq 0. \quad (1)$$

Although  $h$ -field is a set of *Diff*-vectors (or Lorentz-vectors), it corresponds to one irreducible representation of the whole symmetry group (1). This irreducibility is absolutely necessary for our aim – elimination of arbitrary parameters. (The global scale transformations [ $\kappa$  in (1)] entail the notion of [length] dimension [2,3], a global feature too. In absence of the  $\Lambda$ -term, GRT has similar global [conformal] symmetries,  $g_{\mu\nu} \rightarrow \kappa^2 g_{\mu\nu}$ .)

The simplest AP covariant, besides the metric  $g_{\mu\nu} = \eta_{ab} h^a_\mu h^b_\nu$  [ $\eta_{ab} = \text{diag}(-1, 1, \dots, 1)$  is the Minkowski metric], is a tensor composed from the first-order derivatives (see [2,3] for details):

$$\Lambda^a_{\mu\nu} = h^a_{\mu;\nu} - h^a_{\nu;\mu} = h^a_{\mu;\nu} - h^a_{\nu;\mu} = 2h^a_{[\mu;\nu]}; \quad \Lambda^a_{[\mu\nu;\lambda]} \equiv 0. \quad (2)$$

Comas and semicolons denote partial and covariant (Levi-Civita) derivatives respectively.

Einstein and Mayer [1] found many compatible second-order ( $4D$ -)equations, with third-order identities, although the complete compatibility theory appeared well later [4]. For example,

$$\mathbb{A}^a_\mu = \Lambda^a_{\mu\nu;\nu} = 0 \quad (\mathbb{A}^a_{\mu;\mu} \equiv 0) \quad (3)$$

is a simple (non-Lagrangian, no term  $h^a_\mu L$ ) compatible equation. Its linearization looks like a  $D$ -fold Maxwell equation with  $D(D-2)$  polarizational degrees of freedom (*polarizations*).

## 2 Singularities of solutions in AP; the perfect theory

The compatibility test can be extended on 1-degenerated co-frames,

$$h^a_\mu \rightarrow \text{diag}(1, \dots, 1, 0), \quad h = \det h^a_\mu \rightarrow 0; \quad \text{minor } hh^a_\mu = \partial h / \partial h^a_\mu \rightarrow \text{diag}(0, \dots, 0, 1), \quad (4)$$

because the main terms (principle derivatives) can be written, using co-frame minors of co-rank 2, and for one exception, 2, 3 and 1, in a multilinear (in co-frame) form. E.g., eq.(3) takes the form

$$0 = h^2 \mathbb{A}^{a\mu} = -g (g^{\alpha\mu} g^{\beta\nu} - g^{\beta\mu} g^{\alpha\nu}) h^a_{\alpha;\beta\nu} + \dots = h^a_{\alpha;\beta\nu} \partial^2(-g) / \partial g_{\alpha\mu} \partial g_{\beta\nu} + (h'^2); \quad (5)$$

the 2-minors of co-metric, see (5), are bilinear in co-frame 2-minors [2,3] ( $g = \det g_{\mu\nu} = -h^2$ ).

The principle derivatives in (5) keep regularity (a must for compatibility, [4]) for 1-degenerated coframes (4), and this is a covariant indication of (co-)singularity appearance in solutions for eq.(3). (Invariants quadratic in *Diff*-scalars  $\Lambda^a_{bc} = \Lambda^a_{\mu\nu} h_b^\mu h_c^\nu$  go to infinity for degenerated co-frames.)

The same feature (co-singularities) characterizes all other AP equations (including vacuum GR) with the only exception. The reason of uniqueness is: only the skew-symmetric part of this excep-

tional equation (EE) participates in the third-order identity; its multilinear form involves 2- and 3-minors. The EE's symmetrical part requires 1-minors; on co-frame 1-degeneration, only one 1-minor remains non-zero, (4), and EE's regularity breaks [2,3].

Denoting irreducible parts of  $\Lambda$ -tensor,  $S_{abc} = 3\Lambda_{[abc]}$ ,  $\Phi_a = \Lambda_{bba}$ , one can write EE:

$$\mathbb{E}_{a\mu} = L_{a\mu\nu;\nu} - \frac{1}{3}(f_{a\mu} + L_{a\mu\nu}\Phi_\nu) = 0, \quad 2\mathbb{E}_{[\nu\mu]} = S_{\mu\nu\lambda;\lambda} \quad (2\mathbb{E}_{[\mu\nu];\nu} \equiv 0), \quad (10)$$

where  $L_{abc} = \Lambda_{abc} - S_{abc} - \frac{2}{3}\eta_{a[b}\Phi_{c]}$ ,  $f_{\mu\nu} = 2\Phi_{[\mu;\nu]}$ ; like eq.(3), EE is non-Lagrangian.

The other case (contra-singularity) relates to degeneration of a contravariant frame density of some weight (it gives a tri-linear form for the principle derivatives [2,3]):

$$H_a^\mu = h^{1/D^*} h_a^\mu; \quad h_a^\mu = H^{1/(D-D^*)} H_a^\mu \quad (H = \det H_a^\mu); \quad (11)$$

$D^*$  depends on the equation:  $D^*=\infty$  for eq.(3), and  $D^*=4$  for EE ( $D^*=2$  for GR). The inverse transform of (11) is singular if  $D=D^*$  (forbidden dimension, if integer; at this, EE's trace part becomes irregular, the principle derivatives  $\sim\Phi_{\mu;\mu}$  vanish).

For the exceptional equation (10) the nearest possible dimension,  $D=5$ , is especially important because of the next equality (1-minor of contra-density = co-frame):  $H^{-1}H_a^\mu = h_a^\mu$ ; so, contra-singularities simultaneously imply co-singularities of high co-rank, but that's impossible!

The likely interpretation of this observation is: for EE, contra-singularities are impossible if  $D=5$  (perhaps due to some specificity of *Diff*-orbits on the contra-frame-density space).

Nature should choose this EE and  $D=5$  if she abhors singularities. At least this choice gives us a really beautiful and robust theory, the perfect theory (perfectly no arbitrary parameters, solutions do not break on singularities), which will be the focus of the following exposition.

### 3 Polarization degrees in 5D AP; new gravity and cosmology

EE's symmetrical part (with Einstein's tensor  $G_{\mu\nu}$  detached) does not lead to a proper energy-momentum (no positive energy):

$$\mathbb{E}_{(\mu\nu)} + 2g_{\mu\nu}\mathbb{E}_{\lambda\lambda} : \quad G_{\mu\nu} + \frac{2}{3}\Phi_{(\mu;\nu)} + (\Lambda^2\text{-terms}) = 0 \quad (\mathbb{E}_{(\mu\nu);;\nu} \Rightarrow f_{\mu\nu;\nu} = -\frac{1}{2}S_{\mu\nu\lambda}f_{\nu\lambda}); \quad (12)$$

the "linear" second term here means strong influence of the longitudinal polarization on the metric. However, the following prolonged equation looks like a proper forth-order ( $R^{\mu\nu}G_{\mu\nu}$ -) gravity:

$$\mathbb{E}_{(\mu\nu);;\lambda;\lambda} : \quad G_{\mu\nu;\lambda;\lambda} + G_{\varepsilon\tau} \left( 2R_{\varepsilon\mu\tau\nu} - \frac{1}{2}g_{\mu\nu}R_{\varepsilon\tau} \right) = T_{\mu\nu}(\Lambda'^2, \dots), \quad T_{\mu\nu;\nu} = 0; \quad (13)$$

up to quadratic terms (brackets hint  $B$ -tensor here has the symmetries of Riemann curvature tensor),

$$T_{\mu\nu} = \frac{2}{9} \left( \frac{1}{4}g_{\mu\nu}f^2 - f_{\mu\varepsilon}f_{\nu\varepsilon} \right) + B_{[\mu\varepsilon][\nu\tau]}(\Lambda^2)_{,\varepsilon\tau}; \quad (13')$$

the term  $B''$  adds nothing to the 5-momentum and angular momentum.

Eq.(13) can follow also from the least action principle. The “meta-Lagrangian” (or a “weak-Lagrangian”, through N.Kh. Ibragimov; the variation vanishes due to both field equations and their prolongations) is *quadratic* in the field equations (an *automatic* Lagrangian – one field derivative):

$$\mathcal{L} = \mathbb{E}_{(\mu\nu)}^2 - 7\mathbb{E}_{\lambda\lambda}^2 \equiv R^{\mu\nu}G_{\mu\nu} + \frac{1}{9}f_{\mu\nu}^2 + \frac{4}{9}[\Phi_{\mu}(3G_{\mu\nu} - \Phi_{\mu,\nu}) + \Phi_{\nu}\Phi_{\lambda;\lambda}]_{;\nu} + (\Lambda'\Lambda^2, \Lambda^4). \quad (14)$$

The main quadratic terms, after exclusion of covariant divergences (surface terms), look like fourth-order gravity with a *meta-electromagnetism* (*meta* – no gradient invariance); higher terms add to  $T_{\mu\nu}$  only a trivial quadratic contribution,  $\sim B''$ . New gravity emerges as a bonus (no *ad hoc* activity)!

The pseudo-tensor of energy-momentum for (13) is trivial; only conditional conservation laws relating to symmetries (Killing vectors) are possible. This means the gravitation contribution to the “total energy” is negative, total energy is identically zero (the metric and  $f$ -field behave like two sides of one medal). The same is possible in GR but requires fine tuning (the total density should finely equal the critical one). Only  $f$ -covariant (three transverse polarizations in 5D) carries  $D$ -momentum and angular momentum (tangible waves). Other 12 polarizations are imponderable, or intangible; this feature is very strange – how to quantize?

These  $f$ -waves feel only the metric and  $S$ -field, see (12); the last affects only  $f$ -wave *polarization* (‘spin’):  $S_{\mu\nu\lambda}$  cannot enter the eikonal equation. So  $f$ -waves move along usual Riemannian geodesics; and they produce linear instability of three intangible polarizations: using eq.(10) and differentiating identity (2) one can get the next linearized equations:

$$\Lambda_{abc,dd} = -\frac{2}{3}f_{bc,a}; \quad S_{abc,dd} = 0, \quad \Phi_{a,bb} = 0 \quad (\text{also } f_{ab,cc} = 0 = R_{abcd,ee}). \quad (15)$$

The twelve stable polarizations include:  $S$ -tensor holds three polarizations (the dual tensor behaves like a pseudo-EM field),  $\Phi$ -vector holds four (three tangible, rotor polarization and one longitudinal), and there are five gravitational (Weyl) polarizations [3].

EE has non-stationary spherically symmetric solutions. The condition  $f=0$  is a must for this symmetry (and  $S=0$ ); so,  $O_4$ -waves (longitudinal) carry no energy, weight nothing – lack of gravity!

An appropriate cosmological model might look: a single longitudinal wave (SLW) moves ultra-relativistically along the radius, being filled with chaotic waves of all sorts, including unstable ones which can grow during cosmological time to nonlinear levels. The metric inhomogeneity SLW generates can serve as a shallow dielectric waveguide for  $f$ -waves [2] which should have wave-vectors almost tangent to the  $S^3$ -sphere of the wave-front to be trapped inside the shell. The shell thickness can be small for an observer in the  $O_4$ -symmetry center, but in co-moving coordinates it should be very large (Gamma-larger), but still much smaller than the current radius of the shell,  $L \ll R$ .

The Hubble plot in this (kinda anti-Milne) cosmology reads (see [3] and references therein):

$$\mu = \mu_0 + 5 \log[(1+z) \ln(1+z) + O(\Gamma^{-2})] \quad (\text{the relativistic corrections are small}). \quad (16)$$

Melia suggests similar FRW-like model [5] (a sort of “Dark Curvature”; 4D, no Gamma).

All masses ( $\approx$  “bags” or disturbances, thicknesses of  $f$ -waves) are of the same big size  $L$  along the extra-dimension. The new gravity, eq.(13) plus additional constraint (12), can reproduce the Newton’s law on small scales,  $r < L$ , but on larger scales the force decreases more slowly  $\sim 1/r$  [3] (for scales smaller than the shell radius  $R$ ).

#### 4 Topological quasi-charges; one more (5D) interpretation of QM

Quantum mechanics is not a patented property (and not a sacred cow) – just a working math based on few principles. Most of them are simple: Lorentz covariance (if quanta are rare), positive energy and the least action principle, causality. But the superposition principle is fairly weird. Well.

The meadow on the left picture below seems desert, but an invisible person is walking there. If there is raining, she manifests herself (the right picture). The harder rain, the more visible she is!



This toy analogy should illustrate the point: emerging phenomenology of topological quanta (T-quanta, nonlinear field configurations carrying topological quasi-charges; a sort of *topological Brownian particles*) surviving in the stochastic cosmological background (a topological superstructure over the classical substructure) can look as a 4D QFT. The superposition principle *emerges* due to huge size of T-quanta along the extra-dimension and the fact that  $f$ -waves are almost tangential in the shell – scattering amplitudes of quanta’s parts with the same projection (i.e. co-phased) should be summed up providing from eq.(14) a *second-entry accounting*, a quantum 4D-Lagrangian.

Classification of topological quasi-charges [2] (examples of their differential forms are given in [6]) gives few insights on the SM: on the lepton flavors (three or four), quarks (twice as many), neutrinos (true neutral, kinda Majorana); no room for SUSY and DM; no spin zero elementary quanta – only 1 and  $\frac{1}{2}$  [sure, spinor-like terms can emerge only via the first term (not second) in  $\mathcal{L}$  (14)].

And the Planck length here should be a composite value,  $\lambda_{Pl} \sim a_f L$  ( $a_f$  is the amplitude of  $f$ -waves, “tangential, all-round precipitates”), not a fundamental scale.

#### 5 Gravitational waves

Gravitational waves (GWs) are generated essentially by the Ricci tensor (the curvature evolution equation follows from the Bianchi identity) [3]:

$$R_{\mu\varepsilon\nu\tau;\rho;\rho} = R_{\mu\nu;(\varepsilon;\tau)} - (\mu \leftrightarrow \varepsilon) - (\nu \leftrightarrow \tau) + (\text{Riem}^2). \quad (17)$$

Excluding space derivatives (they don't matter in the wave zone), one can obtain using the new gravity equations (with  $\square R_{\mu\nu} \sim T_{\mu\nu}^{(m)}$ ; *mathematical* and *physical* stress-energy tensors are used)

$$\square R_{0\alpha 0\beta} \sim \ddot{R}_{\alpha\beta} \sim T_{\alpha\beta}^{(m)} \quad (L^2 T_{\mu\nu}^{(m)} = \lambda_{\text{Planck}}^2 T_{\mu\nu}^{(\text{ph})}), \quad (18)$$

whereas in GR eq.(10) gives something different:

$$\square R_{0\alpha 0\beta} \sim \ddot{R}_{\alpha\beta} \sim \lambda_{\text{Planck}}^2 \ddot{T}_{\mu\nu}^{(\text{ph})} \sim L^2 \omega^2 T_{\mu\nu}^{(m)}. \quad (19)$$

Assuming the new gravity is correct, one can conclude that GR strongly overestimates, by factor  $(L/\lambda)^4$  where  $\lambda$  is the GW-wavelength, the intensity of short GWs, and strongly underestimates intensity of very long (cosmological scale) GWs.

## References

[1] A. Einstein and W. Mayer, *Systematische Untersuchung über kompatible Feldgleichungen, welche in einem Riemannschen Raume mit Fernparallelismus gesetzt werden können*. Sitzungsber. preuss. Akad. Wiss., phys.-math. Kl, 1931, 257-265. (Einstein's Full Collection. **2**, 353-365. Rus.)

[2] I.L. Zhogin, *Old and new research on the Absolute Parallelism theory*. Lambert Academic Publishing: 2010, ISBN 978-3-8383-8876-2; arXiv: gr-qc/0412130v2.

[3] I.L. Zhogin, *Absolute parallelism, modified gravity, and suppression of gravitational 'short' waves*. Proc. of PIRT-2011. Moscow, 2012. 337-346; arXiv: 1109.1679.

[4] J.F. Pommaret, *Systems of Partial Differentiation Equations and Lie Pseudogroups*. Math. and its Applications, **14**, New York, 1978.

[5] F. Melia, *The  $R_{h=ct}$  universe without inflation*. arXiv: 1206.6527v2

[6] I.L. Zhogin, *Topological charges and quasi-charges in AP*. arXiv: gr-qc/0610076v3